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SEMIAMPLE HYPERSURFACES IN TORIC VARIETIES

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0. Introduction. While the geometry and cohomology of ample hypersurfaces in toric varieties have been studied (see [BC]), not much attention has been paid to semiample (i.e., "big" and "nef") hypersurfaces defined by sections of line bundles generated by global sections with a positive self-intersection number. It turns out that mirror symmetric hypersurfaces in the Batyrev mirror construction [B2] are semiample, but often not ample. In this paper, we study semiample hypersurfaces. Such hypersurfaces bring a geometric construction that generalizes the way of construction in [B2].

The purpose of this paper is to present some approaches to studying the cohomology ring of semiample hypersurfaces in complete simplicial toric varieties. In particular, we explicitly describe the ring structure on the middle cohomology of regular semiample hypersurfaces, when the dimension of the ambient space is 4. Let us explain the main ideas of computing the topological cup product. The first step is to naturally relate the middle cohomology of the hypersurfaces to some graded ring; in our situation this is done using a Gysin spectral sequence. The origin of this idea is in [CarG] and [BC]. The second step is to use the multiplicative structure on the graded ring in order to compute the topological cup product on the middle cohomology. We remark that the cup product was computed on the middle cohomology of smooth hypersurfaces in a projective space (see [CarG]), and this paper generalizes some of the results in [CarG].

The following is a brief summary of the paper. In Section 1, we establish notation and then introduce a geometric construction associated with semiample divisors in complete toric varieties. At the end, we give a criterion for a divisor to be ample (generated by global sections) in terms of intersection numbers. This was known for simplicial toric varieties (the toric Nakai criterion), and we prove it for arbitrary complete toric varieties.

Section 2 studies regular semiample hypersurfaces and describes a nice stratification of such hypersurfaces. These hypersurfaces generalize those in the Batyrev construction (see [B2]).

Section 3 generalizes the results of [CarG] on an algebraic cup product formula for residues of rational differential forms (from here on, the toric variety is usually simplicial). It shows that there is a natural map from a graded ring (the Jacobian ring

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