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Erratum

Erratum for Christian Espíndola, "Semantic Completeness of First-Order Theories in Constructive Reverse Mathematics," *Notre Dame Journal of Formal Logic*, vol. 57, no. 2 (2016), pp. 281–86. DOI 10.1215/00294527-3470433.

The proof of the implication $2 \implies 1$ of Theorem 3 was incorrect. Here is a correct proof.

($2 \implies 1$) Let $\mathcal{B}$ be a Boolean algebra, and consider the theory Γ over a language which has a constant for every element of $\mathcal{B}$ (we shall identify such elements with the constants themselves), a unary relation F, ($F(a)$ is thought of as the assertion "a is in the filter"), and whose axioms are the following:

1. $F(1) \wedge \neg F(0)$;
2. $F(a) \to F(b)$ for each pair $a \leq b$ in $\mathcal{B}$;
3. $F(a) \wedge F(b) \to F(a \wedge b)$ for every pair a, b in $\mathcal{B}$;
4. $F(a) \vee F(\neg a)$ for each a in $\mathcal{B}$.

Since this theory is finitely satisfiable (because every finite subset of $\mathcal{B}$ generates a finite subalgebra where one can construct an ultrafilter), it is consistent, and hence, by hypothesis, there is a model $\mathcal{M}$ with a satisfaction relation $\vDash$. Define now $\mathcal{U} = \{a \in \mathcal{B} : \mathcal{M} \vDash F(a)\}$. It is easy to prove that $\mathcal{U}$ is an ultrafilter of $\mathcal{B}$ (and so it follows that every Boolean algebra has an ultrafilter, which is a well-known equivalent of Boolean prime ideal theorem). Indeed, $1 \in \mathcal{U}$ since $\mathcal{M} \vDash F(1)$, and $0 \notin \mathcal{U}$ since $\mathcal{M} \nvDash F(0)$ by the consistency property. If a and b are in $\mathcal{U}$, then $\mathcal{M} \vDash F(a)$ and $\mathcal{M} \vDash F(b)$, so by soundness we have $\mathcal{M} \vDash F(a \wedge b)$ and hence, $a \wedge b$ belongs to $\mathcal{U}$. If $a \leq b$ and a is in $\mathcal{U}$, then $\mathcal{M} \vDash F(a) \to F(b)$, and since $\mathcal{M} \vDash F(a)$, by soundness we get $\mathcal{M} \vDash F(b)$ and so b is in $\mathcal{U}$. Finally, since for every a in $\mathcal{B}$ we have $\mathcal{M} \vDash F(a) \vee F(\neg a)$, then by the consistency property we have that either $\mathcal{M} \vDash F(a)$ or $\mathcal{M} \vDash F(\neg a)$; that is, either a or $\neg a$ is in $\mathcal{U}$.

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