Corrigendum to "On the homogeneous ergodic bilinear averages with Möbius and Liouville weights" by E. H. el Abdalaoui

There is an error (a gap) in the proof of Theorem 4.5 which affects the proof of Theorem 4.1 and Theorem 4.6. The error occurs in the estimate in line 11 on page 12 for $\mathbb{Z}_{\bar{J}}$ equipped with the probability measure. In this case, the constant on the right-hand side is missing a factor of J in the numerator.

 The proof of the second main result (Theorem 4.2) is completely independent from the proof of Theorem 4.5. This is also the case for the other results in Section 3. Therefore, the following theorem (Theorem 4.2) still stands.

THEOREM
Let $(X, \mathcal{A}, \mu, T)$ be a weakly mixing ergodic dynamical system, and assume that the spectrum of the restriction of T to its Pinsker algebra is singular. Then, for any $f_j \in L^\infty(X)$, $j = 1, \ldots, k$, for almost all $x \in X$, we have

$$\frac{1}{N} \sum_{n=1}^{N} \boldsymbol{\nu}(n) \prod_{j=1}^{k} f_j(T^{jn} x) \xrightarrow[N \to +\infty]{} 0,$$

where $\boldsymbol{\nu}$ is the Liouville function or the Möbius function.

However, the author would like to point out that, as far as he knows, there is no counterexample against Theorem 4.5 as it stands. According to Theorem 4.2, we conjecture that the statement holds.