ERRATUM FOR “HEIGHTS OF VECTOR BUNDLES AND THE FUNDAMENTAL GROUP SCHEME OF A CURVE”

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1. Introduction

Let B be a Dedekind scheme, and denote by K its function field. Let X be a connected scheme over B endowed with a section $x_0 : B \to X$.

In [2, Section 2(b)], the third author claims the proof of the following conjecture.

CONJECTURE 1.1
Let $f : X \to B$ be a faithfully flat morphism locally of finite type with X integral, endowed with a section $x_0 : B \to X$. Then the fundamental group scheme $\pi(X/B, x_0)$ exists.

It is explained in [2, Section 2(b)] that a key step in the proof of the existence of $\pi(X/B, x_0)$ is Property 1.2 below.

Property 1.2
Let G be a finite and flat B-group scheme, and let $E \to X$ be a G-torsor. Suppose that there is a reduction of structure group for the generic fiber $E_K \to X_K$ from G_K to some closed subgroup scheme $H_K \subset G_K$. Then there is a reduction of the structure group for the torsor $E \to X$ itself from G to $\overline{H_K}$, where $\overline{H_K}$ denotes the schematic closure of H_K in G.

Unfortunately, the argument proposed in [2] to prove Property 1.2 contains a mistake. Indeed, Property 1.2 does not hold under these general hypotheses. A counterexample (proposed by Jilong Tong) is described in [1, Section 6].

2. Solution

In our recent paper [1], we prove that Property 1.2 holds in one of the following situations:

(1) $X \to B$ is faithfully flat, locally of finite type, and for all $s \in B$ X_s is reduced;

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