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Liang Xiao has pointed out that the proof of Theorem 4.2.3 is incomplete: it assumes that the differential module N of rank 1 over R' is free, and this is not immediate in general (see Remark 3.1.3). The published proof does correctly reduce Theorem 4.2.3 to the fact that, for N a rank 1 differential module over $R_{n,m}$, there exists $s \in R_{n,m}$ for which $E(-s) \otimes_{R_{n,m}} N$ is regular. We give here an alternate proof of this statement, by induction on m; the base case $m = 0$ is trivial because N is regular (e.g., by Theorem 4.1.4).

Recall (Notation 4.1.1) that $F_{(m)}$ is defined to be the completion of $\operatorname{Frac}(R_{n,0})$ with respect to the x_m-adic norm. If we put $S = R_{n,m-1}/(x_m)$ and $\kappa_{(m)} = \operatorname{Frac}(S)$, we may identify $F_{(m)} \cong \kappa_{(m)}((x_m))$. Let V_0 be any lattice in $V = N \otimes_{R_{n,m}} F_{(m)}$, and put $N_0 = N \cap V_0$; as in Lemma 4.1.2, the $R_{n,m-1}$-module N_0 is finitely generated and torsion free.

By Theorem 2.3.3, there exists $s \in F_{(m)}$ for which $E(-s) \otimes_{F_{(m)}} V$ is regular. Suppose that s has x_m-adic valuation $-h < 0$; write $s = \sum_{i=-h}^{\infty} a_i x_m^i$ with $a_i \in \kappa_{(m)}$. Suppose further that $a_{-h} \notin S$; since S is noetherian and factorial, a_{-h} fails to belong to the discrete valuation ring T obtained by localizing S at some minimal nonzero prime ideal. (More concretely, this ideal is generated by an irreducible polynomial dividing the denominator of a_{-h} in lowest terms.) Since $N_0/x_m N_0$ is a nonzero, finitely generated, torsion-free S-module, $(N_0/x_m N_0) \otimes_S T$ is a nonzero finitely generated, torsion-free T-module, and hence a nonzero finite free T-module. In particular, it cannot be stable under multiplication by an element of $\operatorname{Frac}(T)$ not contained in T. However, $(N_0/x_m N_0) \otimes_S T$ is stable under multiplication by a_{-h} because for $\mathbf{v} \in N_0$, we have $g x_m^{h+1} \partial_m(\mathbf{v}) \equiv -h a_{-h} \mathbf{v} \pmod{x_m N_0}$. This contradiction shows that $a_{-h} \in S$.

By lifting a_{-h} to $R_{n,m}$ and replacing N with $E(-a_{-h} x_m^{-h}) \otimes_{R_{n,m}} N$, we may reduce the value of h; by induction, we reduce to the case where V is itself regular. In this case, by Proposition 2.1.12, $x_m \partial_m$ acts on $N_0/x_m N_0$ as multiplication by some scalar belonging to the subfield of $\kappa_{(m)}$ killed by ∂_i for $i \neq m$. This subfield is precisely k, so we may twist once more to force $x_m \partial_m$ to annihilate $N_0/x_m N_0$. In

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