(largest contemplated) linear model Ω by Jaeckel's method, say using (3.15). Having obtained the resulting $\mathbf{e}(\hat{\beta}_{\mathrm{JW}})$ residuals $\mathbf{e}(\hat{\beta}_{\mathrm{JW}})$ and an estimate of $\hat{\sigma}_{\mathrm{R}}$, form pseudo values for the data as follows,

$$Y_i^* = [X\hat{\beta}_{\mathrm{JW}}]_i + \hat{\sigma}_{\mathrm{R}}\left[R_i(\hat{\beta}_{\mathrm{JW}}) - \frac{(N+1)}{2}\right], \quad i = 1, \cdots, n.$$

The heuristics of Bickel (1976) can I believe be rigorized in this case also to conclude that if we now act as if the Y_i^* were the data, apply ordinary least squares methods in fitting subhypotheses and then calculate the usual F statistics we are asymptotically right in the sense that the asymptotic null distribution and power functions of these statistics agree with the χ^2 approximations to the corresponding Hettmansperger-McKean statistics. We expect more. For instance, application of Tukey's method of multiple comparisons to the pseudo observations should have the same efficiency (say in terms of length of the intervals) with respect to the method applied to the original observations as the Wilcoxon test has to the t test.

Of course the asymptotic χ^2 approximations here too will be inadequate as Draper points out. However, one might hope that the same empirical observations made by Draper continue to hold, viz., using the classical degrees of freedom for F works adequately.

Let me add a caution. As Draper points out what is done here guarantees robustness only against heavy tails. In particular, sensitivity to high leverage points among the $[X\beta]_i$ is not affected. Nor is sensitivity to heteroscedasticity, dependence, transformation of the Y scale, etc. Perhaps the pseudo values could be used as a first step in procedures where the second step fitting method would address these departures and of course, one would then iterate.

It is worth noting that the scope of the methods discussed by Draper has recently been enlarged by Tsiatis (1986) to handle the case of right censoring of the Y_i. It's not clear what happens to the pseudo value-based procedures in this context.

Finally, it is worth remembering that the scope of purely rank-based procedures is much greater than what is suggested by the Kruskal-Wallis, Friedman-Tukey tests. In particular, ranks not rank of residual procedures are appropriate when one considers transformation models of the form

$$h(Y_i) = [X\beta]_i + e_i \quad i = 1, \cdots, n$$

where the e_i are assumed to come from some parametric family but h is an unknown monotone transformation. See Doksum (1987) and Bickel (1986) for example.

I congratulate David Draper on this clear insightful presentation.

ADDITIONAL REFERENCES

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Comment

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Dr. Draper has provided a very nice exposition and review of two rank-based robust methods for fixed effects ANOVA problems. In so doing, he concentrates on (i) the formal structure of the methods and (ii) robust inference based on rank-based analogues of the classical test statististics, where robust inference is taken to mean robustness of validity and efficiency. Given the author's commitment to focus on the R-estimate approach, I would only wish that he had given some emphasis to examples, and in so doing revealed the exploratory data-analytic use of the methods. As far as the focus on rank-based methods goes, I have a pragmatically motivated reservation based on a concern I share with Draper, namely, robust methods are not widely available in the major statistical packages.

As Draper points out, R-estimates comprise just one of three major classes of robust estimates, with L-estimates and M-estimates being the other two, and

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