where $\hat{t}_i - \hat{t}_0$ is the best linear unbiased estimate of the elementary treatment-control contrast $t_i - t_0$ and f is a convex, nondecreasing function. It is interesting to speculate, therefore, as to whether or not A-optimal BTIB designs are optimal over a large class of "rectangular" optimality criteria. To answer this, of course, requires coming up with suitable criteria as indicated in my first comment. Similar questions might be raised regarding optimal row-column designs also.

As a third comment, I would like to thank the authors for including material on Bayesian approaches to the design problem. Such approaches seem fairly natural in this setting because often the control treatment is a standard treatment about which we have considerable prior information whereas the test treatments are new and less is known about them. One's prior knowledge about the control should be incorporated into the design and, as one would expect, Bayesian results indicate the effect is to reduce the number of replications of the control. To my knowledge, existing Bayesian results have been obtained by allowing approximate designs and optimal designs are often approximate designs. Although seemingly a hard problem, exact Bayesian design results would be quite interesting. Are the authors aware of any research in this direction?

In summary, Hedayat, Jacroux and Majumdar are to be thanked for a readable and thorough survey article. It is to be hoped that this article will stimulate further research and such research will answer, among other things, the questions I have raised above.

Comment

A. Giovagnoli and I. Verdinelli

This is a very useful survey of many known results on optimal designs of experiments when one of the treatments is a control. It comprises a wide variety of results and it is impossible to comment on each one of them in detail. We shall pick up some general themes.

The first remark is on the choice of the optimality criteria. The title is actually somewhat misleading, because the only optimal designs that are surveyed in it are A- and MV-optimal ones. A-optimality and MV-optimality certainly appear to have very intuitive and appealing statistical interpretations and, according to the authors, are the most widely studied criteria for this type of experimental design. It is a rather disturbing thought, however, that neither of these criteria takes into account the covariances of the estimated treatment-control contrasts.

Besides, other criteria may be relevant in this context. For pilot experiments when the control is taken to be known and the interest lies in testing whether or not the overall effect of the new treatments is appreciable, we may want to contrast the *average* new treatment effect with the old one and minimize $\text{var}(\sum_i \hat{t}_i/v - \hat{t}_0)$, i.e., min var $\sum_i (\hat{t}_i - \hat{t}_0)$, $i = 1, \cdots, v$. This criterion, which can be easily extended to the case of more than just one control, is also mentioned by Majumdar (1986) and it seems appropriate to call it J-optimality because it reduces to minimizing trace$(JPC_d - P')$, with J the $v \times v$ matrix of all ones. In Giovagnoli and Righi (1985) and Notz (1985), it is shown that certain J-optimal designs are also E-optimal, where E-optimality is defined as minimizing the maximum variance of all the estimated contrasts $\sum_i c_i(t_i - t_0)$ with $\sum_i c_i^2 = 1$, and conversely some sufficient conditions for E-optimality turn out to ensure J-optimality too. Thus although E-optimality does not appear to have a very natural statistical interpretation when there is a control, E-optimal plans may also deserve attention in some cases.

Lastly we would like to stress that in the Bayesian approach, due to the (possibly) different prior assumptions on the test treatments and the control, it is no longer true that designs which are D-optimal for inference on treatment-control contrasts, i.e., which minimize the determinant of the posterior covariance matrix of those contrasts, are always D-optimal for any set of contrasts. Thus in this case it is worthwhile to look at D-optimality too.

J-optimality shares with A- and MV-optimality (and also with E- and D-optimality, and others) the property of being invariant under all relabeling of the test treatments which leave the control unchanged. We believe this invariance under a suitable group to be the key to many results on optimal designs, and in

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