surprise however is a relative concept and readers of McKean (1973) would not be surprised at all!

No doubt readers will see other ways of addressing these problems using perhaps stochastic calculus without benefit of CA or the theory of Wishart distributions (indeed Mr. James of Leeds University has shown me how to use Wishart matrix theory to establish the Clifford-Green result mentioned above). The main purpose of this work has been to initiate the development of CA as an effective tool in the study of random processes, rather than to develop new results. More recently, and with the same motivation, I have been working on the use of CA to derive the statistics of shape diffusions for k-ads with $k > 3$. Here the technical problem is to find effective ways of dealing with sums involving k summands, when k is not fixed beforehand but must be treated as a symbolic quantity. Some progress has been made, but work is not yet complete.

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Comment

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In stochastic geometry as in number theory, it is easy to ask questions that the layman can understand but that the specialist can only answer with difficulty or not at all. Under the older name, geometrical probability, the subject is old, e.g., Buffon's famous problem was invented around the time Buffon was preparing a French version of Newton's "fluxions." I don't know of any ancient and unresolved conjectures like Fermat's but it is easy to give simple-sounding problems that are hard to solve, e.g., the motivating problem of Kendall's theory of shape. How do the shapes of triangles vary when their vertices are independently and uniformly distributed in a fixed rectangle? This problem arises from questions about whether there is too much "collinearity" in sets of points (see Figures 1 and 2). A recent and very readable survey of Kendall's theory has been given by Small (1988).

All but the most mathematically gifted readers will find this paper difficult. Rather more basic details are given in Kendall (1984), but this too is written for mathematicians. I hope the promised book (now in preparation) by Carne, Kendall and Le will make it clear to statisticians, because I'm sure that this is a fascinating area for research and applications. To support this belief I will give a brief summary of my own related efforts, sticking mainly to triangles. This is reasonable because most of the suggested applications use them and they are the simplest case.

The shape of Δ, a triangle P_1, P_2, P_3, with vertex angles α_1, α_2, α_3, could be defined as the pair (α_1, α_2). But for most problems this is not easy to work with, or to generalize to k labeled points in $\mathscr{R}^m$. There are lots of other ways to define the shape of a triangle. We may think of Δ as a 2×3 matrix $[z_1, z_2, z_3]$, where the column z_i has elements x_i, y_i, and denotes the position of the vertex P_i in the plane. Because we are only interested in the shape of Δ we may translate, dilate and rotate Δ without changing the shape of Δ, so we seek a "canonical" triangle. Kendall's approach is a variant of the following. Change the origin to the centroid of the triangle and consider the singular value decomposition of the new 2×3 matrix, $R\Lambda L'$, where R is a 2×2 rotation and so irrelevant. By scaling we could make $\lambda_1^2 + \lambda_2^2 = 1$. The remaining object defines the shape. See Mannion (1988) for a simple description—it is very similar to the next suggestion—and Small (1988).

I found Kendall's reduction hard to understand and considered (in Watson, 1986) two alternatives, which worked well in the simple planar problem I had posed. Move P_1 to the origin (0, 0), move P_2 to (0, 1), which uses up the available transformations, and denote P_3 by z, which then serves to define the shape of Δ. It is natural to take it as a point in the complex plane. The other alternative came from taking z_1, z_2, z_3 as

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