hypothesis $\varkappa = \varkappa_0$ against $\varkappa > \varkappa_0$ because under Central Place Theory, $\varkappa$ will be very large under H_1. Note that under both hypotheses, we have $\ell' = (0, 0, 1)$.

Under the Fisher approximation to $K(\ell, \varkappa)$, we could use under H_0

$$2\gamma_0(n - \Sigma z_i) \sim \chi^2_{2n}, \ \gamma_0^{-1} = \varkappa_0^{-1} - \frac{1}{5}\varkappa_0^{-3},$$

where (x_i, y_i, z_i), $i = 1, \cdots, n$, are the n spherical coordinates for Delaunay's triangles specified on the half-lune as in Kendall (1983). The critical region is the lower tail of the distribution. Note that in terms of Bookstein's shape variables for the triangles (Q_{1i}, Q_{2i}), $i = 1, \cdots, n$, we have

$$z_i = \sqrt{3}Q_{2i}/(Q_{1i}^2 + Q_{2i}^2 - Q_{1i} + 1).$$

There is considerable room to improve the test. For example, we could estimate the percentage points of the test by simulating the Poisson process. Also, we could carry out a test for the non-nested hypothesis of the Miles' distribution versus $K(\ell, \varkappa)$ without any approximation. All these ideas require further investigation. Another approach when the size of the triangles is important is to use the mean area of triangles like Mardia (1977) but now without normalizing to $R = 1$. Its mean and variance are known under the Miles' distribution and thus we can test the null hypothesis. Of course, testing H_0 is only a small part of the main problem; the shape and size summary statistics themselves are revealing, e.g., in investigating comparative evidence of Central Place Theory for various different data. It would be interesting to see a detailed analysis of the Wisconsin data along the lines given in the paper.

Finally, let me say that I found the paper very stimulating and look forward to reading the forthcoming book.

ADDITIONAL REFERENCES

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Comment

Wilfrid S. Kendall

David Kendall has been my close collaborator from the very start of my scientific career, and so it gives me great pleasure to add to the discussion of this paper. I take as my theme the application of computer algebra in statistics and probability. As evidenced from the paper, some of the first instances of this have occurred in the statistical theory of shape. I shall make some remarks on the general application of computer algebra in statistical science, and then turn to the specific application (to the diffusion of shape) with which I have been involved recently.

1. COMPUTER ALGEBRA IN STATISTICS AND PROBABILITY

The reader will have noticed several references to the use of computer algebra (CA) in the investigations reported in the paper. To my knowledge this usage represents one of the first substantial applications of CA in the fields of statistics and probability. The others known to me are my own related work on shape diffusions (referred to in the paper as W. S. Kendall, 1988), which was encouraged by the success of CA in investigating the geometry of shape and is discussed further below; and the work on asymptotics in density estimation as described by Silverman and Young (1987). (I would be most grateful to hear of further instances.)

At present the use of CA in statistical science is in its infancy, although many exciting possibilities beckon. The emergence of readily available and powerful personal workstations gives reason to hope for rapid progress in the next few years. The wide screens, multiple tasking facilities and cut - and - paste editing of these workstations combine to yield a most productive environment for CA.

In what sort of areas might we anticipate CA's profitable employment? At the time of writing it seems

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