the upper margin of the braincase with respect to the lower margin. It is not at all equivalent to the second weakest principal warp (dotted lines, frames (C) and (D)). The bending energy eigenanalysis has extracted these large scale patterns of shape covariance by explicitly weighting empirical covariance patterns inversely to geometric localizability. Other equally plausible geometric patterns, such as bending of the upper or lower structures, are not observed to bear any sample variance.

The example suggests the descriptive possibilities inherent in accommodating the metric geometry of Kendall's shape space to a biological subject matter. One can imagine other modifications of the metric in response to other contexts than the biometric. For instance, one can imagine the statistical study of the positions of a robot arm. When the state of the linkage is coded by the coordinates of its joints, then because certain parts of the robot are rigid, an appropriate measure of "distance" would be somewhat altered from the Procrustes. In another sort of constraint, certain "landmarks" might represent the loci of curves in the data—boundary arcs not otherwise labeled—and would thus be "deficient" by one coordinate; again the Procrustes metric needs to be modified. In a study of schools of fish, or flocks of birds, an appropriate shape metric might be the Cartesian product of a biological shape space by a hydro- or aerodynamic one (for the *V* of migrating geese, for instance). Yet other modifications would arise when the points of Kendall's space are "colored" in classes whose separate patterns cannot be usefully studied without reference to their interpenetration, as in problems of multispecies ecology. These and other possibilities represent an enrichment of the metric geometry of shape space within the global purview pursued so sparely and elegantly by David Kendall.

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ADDITIONAL REFERENCES

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Comment

Christopher G. Small

With a high standard of rigor David Kendall has given us an interesting survey of the theory of shape analysis that he has pioneered with the help of others over the last decade. This work is now of sufficient volume that the many topics discussed in this survey can be only briefly touched upon. I certainly hope that this paper is a stimulus to additional consideration of this topic by statisticians. It may well be that on future occasions the topologists will have to introduce their theory of shape with preparatory remarks to the effect that it is not to be confused with the growing statistical theory of shape.

At first glance, this paper might seem to have much in common with the differential geometric techniques in statistics that are associated with Amari (1985) and others. However, despite the abstraction of some of the theory, the methods of Kendall are essentially data analytic rather than model theoretic: the differential geometry is on the sample space not the parameter space. So how much differential geometry must the data analyst know in order to implement the techniques that are described in this paper? The answer is largely dependent on the amount of software

Christopher G. Small is Associate Professor, Department of Statistics and Actuarial Science, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1.