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Further references will be found in the survey papers by Bookstein (1986), Kendall (1984, 1986, 1989) and Small (1988).

Comment

Fred L. Bookstein

The elegant metric geometry of David Kendall's shape spaces Σ_m^k is inherited from the Euclidean metric of the spaces $\mathbf{R}^m$ containing the original point data. In the applications he has sketched here, the points in $\mathbf{R}^m$ are independent and identically distributed (iid) and the metric in shape space, in turn, is symmetric in the points, a sort of spherical distance. Point data generated in other disciplines, however, are not always iid; different metrics may be appropriate to those applications. In this comment I justify a certain analysis of small regions of Kendall's shape space by using a metric quite different from the usual Euclidean-derived version, depict its relation to Kendall's metric and indicate the sort of inquiries it permits.

Morphometrics is the quantitative description of biological form. Its data can often be usefully modeled as sets of labeled points, or landmarks, that correspond for biological reasons from organism to organism of a sample (Bookstein, 1986). We say that these points are biologically homologous among a series of forms: they have identities—names—as well as locations in some Cartesian coordinate system. Any set of landmark locations has a "size" and a "shape" that may be construed according to Kendall's definitions. But the biological relations among different instances of such configurations partake of a feature space not effectively represented by the metric inherited from $\mathbf{R}^2$ or $\mathbf{R}^3$.

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In the biological context, my style of statistical analysis of shapes proceeds, as Kendall pointed out in 1986, within a tangent space of his Σ_2^k or Σ_3^k in the vicinity of a sample "mean form." (Small (1988) has an interesting comment on this construction.) The questions that in Kendall's applications are asked of an entire shape space—questions about concentration upon the "collinearity set," and the like—are replaced in morphometric applications by the more familiar concerns of multivariate statistical analysis: differences of mean shape, covariances involving shape or factors that may underlie shape variation.

In the linearization of Kendall's shape space that applies to this tangent structure, the natural shape metric is an algebraic transformation of the "Procrustes metric," the ordinary summed squared Euclidean distance of two-point configurations after an appropriate optimizing rotation and scaling. But the Procrustes approach is not flexible enough fairly to represent biological structure within the context of multivariate statistical analysis. If two landmarks are typically close together, like the pupil of the eye and the outer corner, then we expect them to move together in their relation to more distant structures. The half-width and the orientation of the eye are more tightly controlled by diverse biological processes of regulation than is, say, the distance from the eye to the chin. These considerations lead one naturally to search out a shape metric that weights changes in small distances more heavily than changes in larger ones. In 1985, David Ragozin of the University of Washington suggested to me that the formalism of