where the Y_j's and Z are independent Gaussian processes with covariance functions $\sigma_j(x_j - w_j)$ and $\sigma_z(x - w)$ respectively, so that

$$\operatorname{cov}(Y(x), Y(w)) = \sum \sigma_j(x_j - w_j) + \sigma_z(x - w).$$

One specific parametric form of this model that might be worth exploring is

$$\begin{aligned}\operatorname{cov}(Y(x),\ &Y(w))\\ &= \sum C_j(\alpha_j \mid w_j - x_j \mid)^\nu K_\nu(\alpha_j \mid w_j - x_j \mid)\\ &\quad + D \prod (\beta_j \mid w_j - x_j \mid)^\nu K_\nu(\beta_j \mid w_j - x_j \mid).\end{aligned}$$

A large C_j would correspond to an important main effect. The model for $Z(\cdot)$ is somewhat problematic as it allows $Z(\cdot)$ to have an additive component. Following the decomposition into main effects and interactions from Section 6 of the article by Sacks, Welch, Mitchell and Wynn, it might be more satisfying to define $Z(\cdot)$ to be a stochastic process with no additive component:

$$Z(x) = Z^*(x) - \sum_j \int Z^*(x) \prod_{h \neq j} dx_h + (d-1) \int Z^*(x)\,dx,$$

where d is the number of dimensions of x and $Z^*(x)$ is a Gaussian process with some simple covariance function. I think it would be very interesting to find optimal designs under some models of the general form given by (1). If the optimal designs are very different from those obtained by Sacks, Welch, Mitchell and Wynn for their models, that would call into question the effectiveness of their designs for processes where most of the variation can be explained by main effects.

ADDITIONAL REFERENCES

ABRAMOWITZ, M. and STEGUN, I. (1965). *Handbook of Mathematical Functions*, 9th ed. Dover, New York.

YAGLOM, A. M. (1987). *Correlation Theory of Stationary and Related Random Functions* **1**. Springer, New York.

Rejoinder

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We thank the discussants for their incisive comments, suggestions and questions. Nearly all the discussants have been key participants at the workshops mentioned by Johnson and Ylvisaker; all have been instrumental in the development of new methodologies for the design and analysis of computer experiments. Most of the comments and our responses are concerned with the choice of the experimental design and the choice of the correlation function.

We had hoped that the example of Section 6 would attract some suggestions from the discussants, and in this we are not disappointed. Morris' results on the first-stage, 16-point design are interesting—in particular, they indicate that the concentration of the design in the center of the region also occurs for the much rougher process corresponding to $p = 1$ in (9). As this is only a preliminary stage, and there is not much to be lost by using a cheaper design anyway, his scaled quarter fraction makes a lot of sense. In a seven-dimensional problem, Sacks, Schiller and Welch (1989) similarly reduced the optimization problem by restricting attention to scaled central-composite designs. Without doing the optimization or amassing experience from many problems, though, we cannot know when the relative performance of cheap designs will be satisfactory.

For all 32 runs, Easterling recommends two complementary quarter fractions. He rightly points out the advantage of not having to optimize anything, and we tried these fractions on $\{-\frac{1}{2}, \frac{1}{2}\}^6$ and $\{-\frac{1}{4}, \frac{1}{4}\}^6$. In some recent applications where data are cheap to generate, we have been using Latin hypercube designs, and for comparison we also report results for a 32-run Latin hypercube. The six factors have the same 32 equally spaced values, $-\frac{31}{64}, -\frac{29}{64}, \ldots, \frac{31}{64}$, but in different random orders. For both designs, the predictor is based on model (14) after re-estimating the parameters $\theta_1, \ldots, \theta_6$ and p in the correlation function (9). Table R1 shows the average squared error of prediction at the same 100 random points we used previously. For ease of comparison, the results for our original design are repeated. The complementary quarter fractions and the Latin hypercube perform similarly, with our design showing a modest advantage.

It is of interest to note that, for certain values of n and d, scaled standard designs can be optimal. For 8 points in 4 dimensions and 16 points in 5 dimensions