

range of possible choices for A , which allows the construction of good algorithms for many different types of statistical models.

This brings me to the question of choice of algorithm, or, in the case of iterative weighted least squares, the choice of the matrix A . The choice of algorithm depends very much on the expected use of the algorithm, and there is a world of difference between an all-purpose algorithm and an algorithm tailored for a specific application. For example, the Fisher scoring algorithm may be considered a good general algorithm. However, in many specific applications, it is easy to find better algorithms, for example the algorithms based on the deviance weights or score weights mentioned above. Another example is the case of a convex objective function, for which the Newton–Raphson algorithm is the natural choice for a general algorithm. However, if the objective function is close to being nonconvex, as is the case for example for the hyperbolic distribution mentioned in Jørgensen (1984), the Newton–Raphson algorithm may become unstable, and, again, one of the two algorithms mentioned above may offer a more stable performance. An extreme case of this is L_1 -estimation, where the Newton–Raphson algorithm fails, whereas the algorithm with score weights may be used.

Finally, I want to point out that our understanding of the relative performance of algorithms is still, at best, incomplete. I believe that the study of convergence, as practiced in the mathematics of optimization, is a fairly crude and incomplete tool for the understanding of the performance of algorithms, at least for statistical algorithms. For example, I have, until now, never seen a satisfactory explanation of the fact that Fisher's scoring algorithm works extremely

well in the case of generalized linear models, as exemplified by GLIM. I have rarely seen an example of a generalized linear model where the algorithm diverges, in spite of the fact that no steplength calculation is performed (in GLIM), and the number of iterations to convergence is, in the majority of cases, around three to five. This is in contrast to the case of more general, non-exponential, models where the Fisher scoring algorithm may become excruciatingly slow, even when a steplength calculation is included. To draw a parallel, the simplex algorithm for linear programming is known to perform much better in praxis than expected on the basis of a worst-case analysis. Not surprisingly, at least to a statistical audience, a more complete understanding of the effectiveness of the simplex algorithm was obtained only after a probabilistic analysis of the algorithm was performed (cf. Borgwardt, 1987 and references therein). Similarly, I suspect that our understanding of the performance of iterative weighted least-squares algorithms will remain incomplete until a probabilistic analysis of the algorithm has been undertaken.

ADDITIONAL REFERENCES

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Comment

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TERMINOLOGY

del Pino draws a distinction between *iteratively weighted least squares* (IWLS), in which the response vector Y is assumed to have a diagonal covariance matrix V , and *iterative generalized least squares* (IGLS), in which V is an arbitrary covariance matrix. For purposes of exposition this distinction seems rather inconsequential, and, to my mind, insufficient

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to justify the usage of two four-letter acronyms. For numerical purposes, however, the savings in computational effort and organizational overhead resulting from the assumption of independence are very substantial. Thus, as the title suggests, the most useful distinction relates to computational organization rather than to conceptual issues.

ESTIMATING EQUATIONS VERSUS MINIMIZATION CRITERIA

del Pino is correct in his claim that the generalization of Gauss–Markov estimation is most naturally