translation parameter θ, and make use of the Kolmogorov-type statistic $D_n(g, P)$ based on the shell function $g(x) = (1 - |x|^2)^+$. Let $\mathscr{P} = \{P^\theta: \theta \in \mathbb{R}^2\}$ be the translation family of probability measures defined by $P^\theta(A) = P(A - \theta)$. Suppose one wishes to estimate θ by the minimum distance estimator, $\hat{\theta}_n$, defined as that value of $\theta \in \mathbb{R}^d$ which minimizes the distance

$$D_n(g, P^\theta) = \sup_t |P_n g(\cdot - t) - Pg(\cdot - t - \theta)|.$$

Under the assumption that the true parameter is $\theta_0 = 0$, it appears that the asymptotic distribution of $\hat{\theta}_n$ may be the same as that for Pollard's estimate, $\hat{\tau}_n$, the value of t at which $P_n g(\cdot - t)$ is maximized, even though the minimization problems are different. Let me offer as a third test of the author's methodology the question of determining the limiting distribution of $n^{1/2}\hat{\theta}_n$. This type of problem is similar to one considered by Blackman (1955), except that he used a Cramér-von Mises distance rather than a Kolmogorov one; in Pyke (1970) this simpler problem was used to illustrate the applicability of the "weak implies strong" methodology mentioned above.

Although I have directed my comments on the paper towards statisticians as users of this theory, I would stress that the paper is also of great value to those doing research in the area. From both viewpoints I greatly appreciate the efforts of David Pollard for preparing this valuable exposition.

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Comment

Miklós Csörgő and Lajos Horváth

It is a pleasure to congratulate David Pollard on his masterly glimpse into the theory of empirical processes. His artful development here of the technique of Gaussian symmetrization, of the resulting maximal inequalities for Gaussian processes and their application in the empirical process context leaves us no room for comment on his methods, which extend the concept of a Vapnik-Červonenkis class of sets. He demonstrates the efficiency of these methods by use of two motivating, nontrivial asymptotic problems and succeeds very well in conveying the look and feel of a powerful tool of contemporary mathematical statistics.

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There are also other powerful contemporary tools available for tackling asymptotic problems of mathematical statistics. The ones we have in mind are strong and weak approximations (almost sure and in probability invariance principles) for empirical and partial sum processes based on various forms of the Skorohod embedding scheme, or on various forms of the Hungarian construction. The quoted book of Shorack and Wellner (1986) is also an excellent source of information on these methods. For further references on the methods and their applications, we mention the books of Csörgő and Révész (1981), Csörgő (1983), and Csörgő, Csörgő and Horváth [CsCsH] (1986). For an insightful overview of strong and weak approximations we refer to Philipp (1986) (cf. also the review of Csörgő (1984)). Concerning Hungarian constructions, for those who are really interested, the papers of Bretagnolle and Massart (1989), and Einmahl (1989) are most recommended readings.

Here *we* make use of the first problem discussed by David Pollard to *illustrate* what we mean by *strong*