does not apply to the case we are considering here. From these explanations it follows that it would be safe to adopt the following definitions. By the term "true value" of the difference of the yields of two varieties, sown on κ selected plots, we mean a

[41]

number Δ associated with the difference of the observed partial averages $X_i - X_j$ in such a way that the probability P_t of preserving the inequality

$$|X_i - X_j - \Delta| < t\sigma_{x_i - x_j}$$

is greater than

$$1 - \frac{1}{t^2}$$

for all $t > 0$.

We can determine empirically that the difference of partial averages of the plots sampled shows a fair agreement with the Gaussian law distribution. This encourages us to name the true difference in yields of two varieties a number δ associated with the difference of the corresponding partial averages, under the condition that the probability of preserving the inequality

$$T_1 < X_i - X_j - \delta < T_2$$

equals

$$\frac{1}{\sigma'_{x_i - x_j}\sqrt{2\pi}} \int_{T_1}^{T_2} \exp\left(-\frac{t^2}{2\sigma'^2_{x_i - x_j}}\right) dt,$$

where,

$$\sigma'^2_{x_i - x_j} = \frac{m - \kappa}{m(\kappa - 1)}\left[\sigma_i^2 + \sigma_j^2 + \frac{2\kappa r}{m - \kappa}\sigma_i\sigma_j\right]$$

and $T_1 < T_2$ are arbitrary numbers. [A misprint (or inconsistency) in the preceding equation has been eliminated; cf. formulas (16) and (17).]

We should remember, however, that this definition is not properly justified.

Of course everything that has been said about the comparison of varieties applies to the comparison of fertilizers.

[42]

Comment: Neyman (1923) and Causal Inference in Experiments and Observational Studies

Donald B. Rubin

Dorota Dabrowska and Terry Speed are to be most warmly thanked for bringing this fundamentally important but previously recondite early work of Jerzy Neyman to the attention of the statistical community. It is an honor to be asked to discuss this document, which reinforces Neyman's place as one of our greatest statistical thinkers and clarifies the debt all modern statisticians interested in causal inference owe to Jerzy Neyman. There are several specific contributions in this article (hereafter referred to as Neyman, 1923) that I feel are particularly noteworthy. To delineate these for my discussion, I first provide a brief summary using a mix of Neyman's notation and more standard current notation. I then discuss Neyman's original definition of causal effects in randomized experiments, extensions of it to experiments with interference between units and versions of treatments, and further extensions to observational studies. Three other contributions in Neyman (1923) are also analyzed: his proposal for the completely randomized experiment, his proposal for using repeated-sampling evaluations over randomization distributions, and his specific results on variance estimation in the completely randomized experiment. Throughout, I attempt to relate these contributions of Neyman's to proceeding and contemporary work of R. A. Fisher and others, and to subsequent work, including my own cited in the Dabrowska and Speed introduction. My conclusions regarding the relationship of Neyman (1923) to other work are briefly summarized in the final section.

1. AN OVERVIEW OF NEYMAN (1923)

Neyman begins with a description of a field experiment with m plots on which v varieties might be applied: "$\cdots$ U_{ik} is the yield of the ith variety on the kth plot"; U_{ik} is a "potential yield" (Neyman's term) not an observed yield because i indexes all varieties and k indexes all plots, and each plot is exposed to only one variety. Throughout, the collection of poten-

Donald B. Rubin is Professor and Chairman, Harvard University, Department of Statistics, Science Center, 1 Oxford Street, Cambridge, Massachusetts 02138.