Rejoinder

Glenn Shafer

I am honored that *Statistical Science* has published my inaugural lecture, together with thoughtful comments from a very distinguished group of readers. These readers include Ian Hacking, who is probably the most prominent living philosopher of statistics, and six prominent members of leading statistics departments. I want to thank the editor, Carl Morris, for recruiting these readers, and I want to thank them for the thought and care they have devoted to reading and discussing my lecture.

My chair in the School of Business at the University of Kansas was endowed by Ronald G. Harper, a Tulsa businessman who designs and sells expert systems that combine statistical and artificial intelligence methods to help retailers locate stores, control inventory and set prices. In appointing me to this chair, the university was recognizing my work on probability in expert systems. My first thought, when I was asked to acknowledge the appointment by giving a lecture to a general audience, was that I should discuss the relation between probability and statistics on the one hand and artificial intelligence on the other. I soon realized, however, that my audience knew so little about these topics that I could not cover them all. Much of the audience would not even be aware that some universities have whole departments devoted to statistics. So I left out artificial intelligence. I talked about the intellectual history of probability and statistics and the why and how of statistics departments. I closed with some suggestions for reinvigorating statistics departments.

It takes some chutzpah for a business professor from the provinces, who has not taught in a statistics department for 14 years, to give prescriptions for reforming statistics, and I think it is a tribute to the openness of our discipline that my ideas have been taken seriously by prominent statisticians.

In their comments, the statisticians have focused on the role of applied statistics. These comments seem very important to me, for applied statistics is crucial to the future of our discipline, and the discussants have the stature to help shape this future. In this rejoinder, I hope to raise some questions that will encourage yet further discussion.

I will begin by responding to Ian Hacking's critique of my sketch of the history and philosophy of probability. Then I will turn to the central questions we should ask about applied statistics: What is its intellectual content, and how can we raise its perceived stature *vis-à-vis* the mathematics of statistics?

THE HISTORY AND PHILOSOPHY OF PROBABILITY AND STATISTICS

Ian Hacking's work on the philosophy and history of probability and statistics has been an inspiration to me since my graduate work in the early 1970s, and I have prized his personal encouragement of my own work in these fields. I am honored, therefore, by his serious and extended response to my lecture.

Hacking quite rightly takes me to task for my simplifications of history. It is true that we cannot compress the rise of frequentism into the years 1842 and 1843, that positivism was not a simple phenomenon and that hints of frequentism can already be found in Laplace and even more clearly in Poisson. He has passed over in silence some other equally egregious simplifications. I should mention in particular my simplified account of David Hilbert's ideas on the foundations of mathematics, for several people have protested to me privately that this account comes closer to popular caricature than careful analysis. (For careful analysis, see Benacerraf and Putnam, 1983.)

Hacking's most important point is that the rise of frequentism had complex roots, going beyond positivism. He sees the early nineteenth century's emphasis on measurement as something broader than positivism, and he points to the proliferation of published statistics as a source of frequentism. I find myself only half convinced by his arguments on these points. Why did people in the nineteenth century want to measure frequencies instead of beliefs? And why did statistics on suicides seem stable? The Ian Hacking of 15 years ago would have seen here the influence of theory on desires and perceptions and, intellectual fashion notwithstanding, I think that theory is still part of the story. A complete understanding of the rise of frequentism must take into account the mathematical possibilities of probability theory and the internal logic of the mathematician's drive to apply this theory. Bernoulli's motivation for the law of large numbers was his desire to apply probability theory to social and economic problems, and this same motivation encouraged nineteenth-century frequentism. Probability theory seems to have a very narrow scope of application if every probability must be both a frequency and a belief, as the probabilities in fair games are. It has a much wider scope of application if every frequency is a probability.

I agree with Hacking that probability is similar to many other concepts, in that it has unified prototypes