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The Unity and Diversity of Probability

Glenn Shafer

1. INTRODUCTION

Mathematical probability and its child, mathematical statistics, are relative newcomers on the intellectual scene. Mathematical probability was invented in 1654 by two Frenchman, Blaise Pascal and Pierre Fermat. Mathematical statistics emerged from the work of the continental mathematicians Gauss and Laplace in the early 1800s, and it became widely useful only in this century, as the result of the work of three Englishmen, Francis Galton, Karl Pearson and R. A. Fisher.

In spite of these late beginnings, probability and statistics have acquired a dazzling range of applications. Inside the university, we see them taught and used in a remarkable range of disciplines. Statistics is used routinely in engineering, business, medicine and every social and natural science. It is making inroads in law and in the humanities. Probability, aside from its use in statistical theory, is finding new applications in engineering, computer science, economics, psychology and philosophy.

Outside the university, we see probability and statistics in use in a myriad of practical tasks. Physicians rely on computer programs that use probabilistic methods to interpret the results of some medical tests. The worker at the ready-mix company used a chart based on probability theory when he mixed the concrete for the foundation of my house, and the tax assessor used a statistical package on his personal computer to decide how much the house is worth.

In this article, I will sketch the intellectual history of the growth and diversification of probability theory. I will begin at the beginning, with the letters between the Parisian polymath Blaise Pascal and the Toulouse lawyer Pierre Fermat in 1654. I will explain how these authors, together with James Bernoulli, Abraham De Moivre and Pierre Simon, the Marquis de Laplace, invented a theory that unified the ideas of belief and frequency. I will explain how this unity crumbled under the assault of the empiricist philosophy of the nineteenth century, how the frequency interpretation of probability emerged from this assault and how a subjective (degree of belief) interpretation re-emerged in this century. I will discuss how these intellectual movements have supported the amazing diversity of applications that we see today.

I will also discuss the future. I will discuss the need to reunify the theory of probability, and how this can be done. Reunification requires, I believe, a more flexible understanding of the relation between theory and application, a flexible understanding that the decline of empiricism makes possible. I will also discuss the institutional setting for reunification: departments of statistics. Departments of statistics have been the primary vehicle for the development of statistical theory and the spread of statistical expertise during the past half-century, but they need new strategies in order to be a source of innovation in the twenty-first century. We need a broader conception of probability and a broader conception of what departments of statistics should do.

2. THE ORIGINAL UNITY OF PROBABILITY

In this section, I will sketch how the original theory of probability unified frequency, belief and fair price. (For details, see Hald, 1990; Daston, 1988; Hacking, 1975.)

In order to understand this unity, we must first understand a paradox. The original theory of probability was not about probability at all. It was about fair prices.

Probability is an ancient word. The Latin noun *probabilitas* is related to the verb *probare,* to prove. A probability is an opinion for which there are good proofs, an opinion that is well supported by authority or evidence.

Pascal and Fermat did not use the word probability in their 1654 letters. They were not thinking about probability. They were thinking about fair prices.

Here is the problem they were most concerned with, a problem that had been posed in arithmetic books for centuries, but that they were the first to solve correctly. You and I are playing a game. We have both put \$5 on the table, and we have agreed that the winner will get all \$10. The game consists of several rounds. The first player to win three rounds wins the game. I am behind at the moment—I have won one

Glenn Shafer is Ronald G. Harper Distinguished Professor of Business, School of Business, University of Kansas, Lawrence, Kansas 66045.

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