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Many different types of bootstrap confidence intervals have appeared in the literature. There is now an urgent need for a unified account of the developments and for comparative studies which can provide some recommendations. This article is very timely in this sense. We do agree that the so-called percentile method is currently the most popular one although it is probably the worst one. The article should also serve as an excellent reference for explicit terms (especially the n^{-1} terms) in some commonly needed Edgeworth expansions.

We would like to concentrate our discussion on two topics: the shortest length bootstrap confidence intervals and Efron's bias correction. The idea of shortest length bootstrap confidence intervals is an important one, which up to this point has not received the attention it deserves. Hopefully, the present paper will help in giving it fair treatment. The increased accuracy due to the phenomenon of one-term Edgeworth correction by the bootstrap is well understood in the case of one-sided intervals which can be used for one-sided testing of hypotheses. However, the situation becomes quite complex in the case of two-sided intervals. The asymmetric intervals based on a bootstrap histogram of Studentized statistics do reflect the asymmetric shape of the true sampling distribution by making correction for skewness. The impact on the total coverage or the length of the intervals depends on population parameters and it seems no definite conclusions can be reached. Hall's Table 1 does shed light on this important issue. Therefore, the shortest bootstrap confidence intervals (which need no analytic work) seem to be attractive alternatives in this case. The present article also reports a very surprising finding on these intervals, which is the simultaneous gain in the coverage probability and the reduction in the length. Some intuitive explanations would be desirable.

For those who are used to looking at bootstrap through Edgeworth expansions, the article presents a very clear view of Efron's bias correction and of the accelerated bias correction (abc). The interpretation of the acceleration constant in terms of skewness brings out the connection between the abc and the one-term Edgeworth correction. If second-order correctness is the objective, then it is not clear why one would want to use the abc (which does seem to need some analytic work) instead of simply bootstrapping a Studentized statistic. Generally, an estimator of the standard error is available for the purpose of Studentizing. Furthermore, if we consider third-order correctness, the bootstrap approximation for the Studentized statistics is better than the abc, due to a phenomenon referred to as a partial correction in Liu and Singh (1987) (cf. Remark 2). In this context, we would like to bring to the reader's attention one-step modified pivots such as the modified t-statistics

$$t_1 = t + \frac{(2x^2 + 1)}{6\sqrt{n}} \frac{\hat{\mu}_3}{s_n^3}$$

discussed in Abramovitch and Singh (1985), whose Edgeworth expansion starts