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According to Dudley's (1978) Theorem 7.1, the last term of (12) is  $o_p(n^{-1/2})$ . Next turn attention to the fourth term and note that

(13) 
$$\begin{aligned} \operatorname{Var} \big( E_{\overline{X}^*} \big\{ G_P \big\{ \big[ -\sigma t, \, \overline{X}^* - St \big] \big\} \big) \big) \\ & \leq \operatorname{Var} \big( G_P \big\{ \big[ -\sigma t, \, \overline{X}^* - St \big] \big\} \big) \\ & = \operatorname{Var} \big( E \big\{ G_P \big\{ \big[ -\sigma t, \, \overline{X}^* - St \big] \big\} | \, \overline{X}^*, \, S \big\} \big) \\ & + E \big\{ \operatorname{Var} \big( G_P \big\{ \big[ -\sigma t, \, \overline{X}^* - St \big] \big\} | \, \overline{X}^*, \, S \big) \big\} \end{aligned}$$

in view of the conditional variance formula. The first term of (13) is 0 because  $G_P$  is a mean 0 process, while the second term is less than  $E\{|F(-\sigma t) - F(\overline{X}^* - St)|\} = o(1)$ . Therefore, in view of (9) and (12),

$$P\left(\frac{\overline{X}-Z}{\sigma} \le t\right) - P^*\left(\frac{\overline{X}^*-Z^*}{S} \le t\right)$$

$$= F_n(-\sigma t) - F(-\sigma t) + \overline{X}F'(-\sigma t) + (\sigma - S)tF'(-\sigma t) + o_p(n^{-1/2}),$$

which is of exact order  $n^{-1/2}$  in probability as a consequence of the central limit theorem.

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Peter Hall's paper gives a welcome and illuminating comparison of competing bootstrap confidence intervals for a one-dimensional parameter. Though important, this one-dimensional case is very special in several respects. Techniques such as Studentizing or accelerated bias correction do not generalize readily to confidence sets for a multidimensional parameter. I will address two problems: (i) how to construct analogs of second-order correct bootstrap confidence sets when the parameter  $\theta$  is vector-valued or infinite-dimensional and (ii) how the general approach for multidimensional  $\theta$  relates to the one-dimensional methods discussed by Hall.

1. Consider the following setting: The sample  $x_n$  has distribution  $P_{n,\lambda}$  which depends upon an unknown parameter  $\lambda$ ; the dimension of  $\lambda$  may be infinite: Of interest is the parametric function  $\theta = T(\lambda)$ , which need not be scalar-valued. Let  $R_n(\theta) = R_n(x_n, \theta)$  be a confidence set root for  $\theta$ —a real-valued function of the sample and of  $\theta$ . Let  $J_n(\cdot, \lambda)$  denote the left-continuous cdf of  $R_n$ . Suppose