P. M. ROBINSON

London School of Economics

Martin and Yohai's paper is a fine technical achievement, developing an interesting tool, the influence functional, for describing an aspect of time series behaviour, and continuing the authors' work on the difficult and important problem of time series analysis in the presence of outliers. I have two points, one being a suggestion prompted by their discussion of hypothesis testing, and motivated by the need for test statistics with both good robustness and good power properties against given alternatives. My first and main point concerns Martin and Yohai's approach towards dealing with the outlier behaviour described by their general replacement model, and to some extent this impacts on the use of their influence functional.

Martin and Yohai's general replacement model (2.2) is indeed "general," and even in the pure replacement (PR) and additive outliers (AO) special cases it presents an identifiability problem to which GM and RA rules need not necessarily provide a useful solution. The non-Gaussian character of y and the nonlinear character of the GM and RA rules severely hinders a proper analysis of the identifiability problem. While Martin and Yohai's results embrace w and v with no moments, even bad contamination can be modelled by w and v with finite variance, in which case, if their core x process is indeed "usually Gaussian," a second moment analysis may gain some insight into the identifiability problem in the LS case, and conceivably also into the possible impact of GM and RA estimators on the problem. Denoting means by m_x, etc., and lag-j autocovariances by $c_x(j)$, etc., for the PR model

(1) $$m_y = m_x + (m_w - m_x)m_z,$$

(2) $$c_y(j) = (1 - m_z)^2 c_x(j) + m_z^2 c_w(j) + \{(m_w - m_x)^2 + c_x(j) + c_w(j)\}c_z(j).$$

For the AO model with v independent of x (as assumed by Martin and Yohai in Section 5)

(3) $$m_y = m_x + m_v m_z,$$

(4) $$c_y(j) = c_x(j) + m_z^2 c_v(j) + \{m_v^2 + c_v(j)\}c_z(j).$$

Note that x's ARMA coefficients are functionals of the $c_x(j)$.

It is easily seen that the $c_x(j)$ can be quite unrecognisable from the $c_y(j)$, leading in general not only to inconsistent estimation of x's ARMA coefficients but also to incorrect order determination via criteria such as AIC. Can robustification alleviate these problems? Note that c_y is determined not only by c_x and c_w or c_v, and the frequency of contamination m_z, but also by c_z. We cannot choose $m_x = m_w$ or $m_v = 0$ (thereby eliminating $(m_w - m_x)m_z$, $(m_w - m_x)^2 c_z(j)$, $m_v m_z$, and $m_v^2 c_z(j)$ from (1)–(4), respectively) without loss of generality because without further information only y can be mean-corrected; substantially different m_x and m_w, or nonzero m_v (by no means unlikely, it