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We thank the discussants for their interesting comments and contributions, and the editors and referees for considerable efforts that led to many improvements in this work. We must also thank the intrepid reader, if he or she is still with us, for weathering his or her way through this long article. The many questions given at the end of the paper and the ideas and issues raised by the discussants, indicate (happily) that this is an active area of research.

The discussants address a wide variety of issues in considerable detail. We try to address their comments and questions below. Before addressing each discussant in turn, we would like to present our views on several topics raised collectively by some.

1. The Bayesian paradigm. It seems that our silence about the Bayesian side of smoothing was so loud that it called for equally loud corrective measures from several discussants. *Cox, Kohn and Ansley, Chen, Gu and Wahba* and *Eubank and Speckman* remind us how useful the Bayesian paradigm can be for developing inferential procedures and algorithms. However, in the absence of a repeated sampling or subjective probability justification for the prior, the Bayesian framework is just a heuristic. In such cases, inferences derived from the Bayesian model must be justified through their sampling properties.

There are of course examples where the assumption of a random function has ample justification and where the prior represents a useful frequentist modeling assumption. This is usually called the stochastic process interpretation of the underlying function. For example, the Yates (1939) random effects model for incomplete block designs (we thank Dr. Peter Green for bringing our attention to this area) can be cast as a semiparametric regression model [Green (1985) and Green, Jennison and Seheult (1985)]. Here the "smoother" for fitting the random incomplete block effects is generated by a natural (noninformative) prior. More informative priors allow for spatial trends of various complexity. Wilkinson, Eckhert, Hancock and Mayo (1983) and the many discussants give a useful overview of this important area. If the assumption of an underlying random