This algorithm is a fast and parsimonious way for representing interaction. For example, if, in their spline bases, f and g have p degrees of freedom, then the minimizing product fg has about p degrees of freedom in it. One adds more multiplicative terms until there is no significant decrease in RSS. Furthermore, the multiplicative terms are easy to interpret.

Unfortunately, numerical results indicate that in the nonindependence case, there are a number of local minima in addition to the global minimum. The algorithm always converges, but it may not converge to the global minimum. This makes the selection of a good starting point important. Our experimental results have been that if we use the starting point given by assuming independence, then the iterates have always converged toward the global minimum.

I am currently working on straightening up the details of this representation of bivariate interaction and hope to go public soon.

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We must begin by thanking the authors for a thought-provoking work. As is well known [Kimeldorf and Wahba (1971) and Wahba (1978)], quadratic penalized likelihood estimates (with nonnegative definite penalty functionals) are Bayes estimates. Let $\mathbf{y} = \mathbf{g} + \boldsymbol{\varepsilon}$ with $\mathbf{g} \sim N(0, \Sigma)$ and $\boldsymbol{\varepsilon} \sim N(0, \sigma^2 I)$, then

$$\hat{\mathbf{g}} = \Sigma(\Sigma + \sigma^2 I)^{-1}\mathbf{y} = A\mathbf{y}, \quad \text{say},$$

which also minimizes $(1/\sigma^2)(\mathbf{y} - \hat{\mathbf{g}})'(\mathbf{y} - \hat{\mathbf{g}}) + \mathbf{g}'\Sigma^+\mathbf{g}$, the resulting smoother matrices are all symmetric nonnegative definite with their eigenvalues in $[0, 1)$. This generalizes to the case where Σ is improper, which gives eigenvalues $+1$.

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