ASYMPTOTIC LOGNORMALITY OF *P*-VALUES

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We are indebted to Professor R. R. Bahadur for pointing out that the expressions given in Tables 1 and 2 for the variances of the limiting distributions of the one and two sample *t* test log *P*-values are incorrect. The correct variance for the one sample test is $\theta^2(1+\theta^2)^{-2}(1+\frac{1}{2}\theta^2)$; the correct variance for the two sample test is $\lambda\bar{\lambda}\theta^2(1+\lambda\bar{\lambda}\theta^2)^{-2}(1+\frac{1}{2}\lambda\bar{\lambda}\theta^2)$. These formulas are available by inspection from the expansions of $L_n^{(2)}$ in Bahadur (1960) and are in agreement with the influence functions given in Lambert (1981). The formula in Table 2 for the variance of the permutation test is also in error. The correct formula, which agrees with the influence function given in Lambert (1981), is $\lambda V(\lambda)+\bar{\lambda}V(\bar{\lambda})$ where $V(\lambda) = \text{var}[-\log(\lambda+\bar{\lambda}\exp(\lambda Z-\theta^2/2))]$ where Z is a normal (0,1) rv. A corrected Figure 5 would still have the .025 and .975 fractiles for the permutation *P*-value inside the .025 and .975 fractiles for the normal scores *P*-value, but not by as much. For $0 < m < 1$ the fractiles for the two tests agree to 4 decimals, for $m < 2.125$ they agree to 3 decimals, for $m < 2.625$ to 2 decimals and for $m < 3.5$ to one decimal. Calculations for larger m were not carried out.

We are also grateful to Professors T. K. Chandra and J. K. Ghosh for the following references concerning some of the considerations of Section 3. Expansions of $A = -\log \alpha$ that are more refined than our expansion (1) are given by Chandra and Ghosh (1978) and Chandra (1980) under general conditions on T_n which are stronger than those sufficient for asymptotic lognormality of the *P*-value. Refinements and generalizations of (1) are given also by Groeneboom (1980), Groeneboom and Oosterhoff (1981), Kallenberg (1978, 1981) and Chandra and Ghosh (1982) in certain specific cases.

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