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BOOK REVIEW

ULRICH KRENGEL, *Ergodic Theorems*, de Gruyter Studies in Mathematics, volume 6, de Gruyter, Berlin, 1985, viii + 357 pages, \$49.95.

REVIEW BY ROBERT SINE

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Ergodic theory was born in a flurry of papers in the *Proceedings of the National Academy of Sciences* in 1931–1932. The mathematical setup was a normalized measure space (Ω, F, μ) and a one parameter group $\{\phi_t\}$ of measure perserving maps of Ω. For a subset A in F we can consider the average amount of time that a point spends in A when transported by the dynamics at hand $\{\phi_t\}$. This mean sojourn time takes the form

$$\lim(1/T)\int_0^T 1_A(\phi_t\omega)\,dt.$$

Since the time of Boltzmann, physicists hoped to show the equality of the average time spent in the set A and the size of the set $\mu(A)$. B. O. Koopman (who received his degree from Birkhoff in 1926 and would go on to write joint papers in ergodic theory with both Birkhoff and von Neumann) published a note in 1931 where he observed that the group of measure isometries induces a group of unitary isometries of L_2 and he suggested relating the spectral properties of the generator of that unitary group with properties of the Hamiltonian system flowing in the background. von Neumann learned of this from Koopman in the spring of 1930 and, based on Koopman's observation, proved that the limit in question existed in the L_2 sense and that the limiting value did indeed have the hoped for constant value provided the system was "ergodic"—that is, the only invariant measurable sets were of measure 0 or 1. von Neumann's paper was not communicated until December 10, 1931, but he informed Birkhoff of his result in a letter in October of that year in which he suggested that a.e. convergence may in fact hold. This Birkhoff was able to show by devising a maximal lemma. The Birkhoff proof, described by Halmos [10] as maximally confusing, has been reworked by Hopf, Khintchine, Riesz and Garsia.

Two elements played the major roles in this initial work—the existence of limits of averaged quantities and the identification of these limits. It is these two elements of ergodic theory that are the focal points of the book under review.

We will give a chapter-by chapter outline and then some general comments.

Chapter 1. Measure preserving and null preserving point mappings (70 pages). Garsia's elementary proof of the maximal lemma is used to obtain the almost everywhere theorem. Induced transformations, recurrence and the Hopf decomposition of the transformation into conservative and dissipative parts are developed. One of the first applications of ergodic theory was to limit theorems of stationary processes made by Doob, Hopf and Khintchine in 1934.

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