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BOOK REVIEW

M. J. SHARPE, *General Theory of Markov Processes*, Academic, San Diego, 1988, 410 pages, \$49.50. ISBN: 0-12-639060-6

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Sharpe's book provides researchers with the first comprehensive presentation of the melding of Markov processes with the general theory of stochastic processes and will serve as a standard of style for the emerging generation of students of Markov process theory. The title of the book is a *double entendre*: "General Theory" signifies not only the foundations of Markov processes, but also the "general theory of stochastic processes," a subject which has assumed a central role in modern probability and whose development can be traced largely through the *Séminaire de Probabilités* series. Explaining the ingredients in the meld consumes several paragraphs and touches several major lines of thought in the last four decades of probability theory.

Markov processes. Occasional readers of the Markov process literature may be puzzled by the profusion of seemingly different Markov processes which appear. There are Feller processes, Hunt processes, standard processes, Ray processes and right processes. All of these processes are simply variations of one basic structure. It consists of a measurable space $(\Omega, \mathscr{F})$, a family of functions X_t mapping Ω into a state space E and a family $(P^x)_{x \in E}$ of probability measures on $(\Omega, \mathscr{F})$. The process X_t generates a filtration $(\mathscr{F}_t)$, namely, the appropriate completion of $\sigma\{X_s: s \le t\}$. Finally, there is a shift operator θ_t: $\Omega \to \Omega$ which is tied to the process by the property $X_t \circ \theta_s = X_{t+s}$. These objects constitute a basic Markov process sextuplet $X = (\Omega, \mathscr{F}, \mathscr{F}_t, X_t, \theta_t, P^x)$ if the strong Markov property is satisfied: for every $(\mathscr{F}_t)$-stopping time T, for every positive $\mathscr{F}$-measurable random variable F and for every positive $\mathscr{F}_T$-measurable random variable G, we have

$$E^x[G \cdot F \circ \theta_T; T < \infty] = E^x\left[G \cdot E^{X(T)}[F]; T < \infty\right].$$

All of the processes mentioned above have the property that $t \to X_t$ is right continuous almost surely. Additional assumptions are peculiar to the individual processes. For example, Feller and Ray processes assume certain continuity properties about the associated semigroup P_t, resolvent U^α and state space E. Hunt processes are characterized by the quasi-left-continuity property, namely, $X_{T(n)}$ converges to X_T almost surely whenever $T(n)$ is a sequence of $(\mathscr{F}_t)$-stopping times increasing to T. With the exception of right processes, all of these variations existed in the 1950s. Right processes were introduced later by Meyer (but let us not get ahead of the story).

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