The Annals of Probability
1996, Vol. 24, No. 3, 1647–1652

BOOK REVIEW

MARK A. PINSKY, *Lectures on Random Evolutions*. World Scientific, Singapore, 1991, 150 pages, $36.00.

REVIEW BY JOSEPH C. WATKINS

University of Arizona

In mathematical terms, an *evolution* is extraordinarily simple to state:

$$\dot{x} = \phi(x). \tag{1}$$

In the absence of any context for (1), little of value can be added. Any situation in which ϕ is random can make a rightful claim for being called a *random evolution*. As before, unless we have some additional structure, little in the way of analysis can be performed.

On the other hand, large areas in the theory of probability and random processes begin with a suitable form and interpretation of ϕ. Perhaps the most well known choice is

$$\phi(x, \dot{w}) = b(x) + \sigma(x)\dot{w}. \tag{2}$$

Here, x and $b(x)$ are vectors of the same dimension, say n, and $\dot{w}$ is a vector perhaps of a second dimension, say d. Finally, $\sigma(x)$ is an $n \times d$ matrix. We begin the discussion of stochastic differential equations by inserting a d-dimensional white noise for $\dot{w}$. Nowadays, we study two interpretations of (2): the Itô formulation and the Stratonovich formulation. Excellent books from all four corners of the globe have been written on stochastic differential equations.

A second choice for ϕ is $\phi_t(x_t) = A_t x_t$. In this situation, the index t for the random process is frequently the non-negative integers, and the dot is taken to be the difference

$$\dot{x}_t = x_{t+1} - x_t. \tag{3}$$

Thus, we are studying the product of random square matrices

$$x_{t+1} = (A_t + I) \cdots (A_0 + I) x_0. \tag{4}$$

For the case that $\{A_t; t \in \mathbf{N}\}$ forms a stationary sequence of random matrices, this line of investigation has lead us to an ergodic theory of matrix products and, in particular, to the Oseledec multiplicative ergodic theorem with its companion Lyapunov exponents.

The probabilistic techniques used in the development of a theory of stochastic differential equations and a theory of random matrix products are quite distinct. However, these two topics cover common ground in their

Received March 1996.