

BOOK REVIEW

D. W. STROOCK, *Probability Theory. An Analytic View*. Cambridge University Press, 1993.

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“In fact, I myself enjoy probability theory best when it is inextricably interwoven with other branches of mathematics and not when it is presented as an entity unto itself.” True to this statement in the preface, Stroock has written a very interesting, distinctly personal volume on probability theory with a strong analytic flavor. The book covers many standard topics in a first graduate course on probability as well as a number of more advanced probabilistic subjects, but also contains several deep excursions into pure analysis. Broadly speaking, the analytical results presented in the book either have significant probabilistic ingredients in their proofs or exhibit illuminating analogies to certain probabilistic results.

Here is a brief description of the book’s contents. Chapters I and II deal with limit theorems for sums of independent random variables. Weak convergence, Wiener measure and Lévy processes are presented in Chapter III. Chapter IV is devoted to a deeper study of Wiener measure. Chapters V and VI present discrete parameter martingales and various applications. Continuous martingales are discussed in Chapter VII. Finally, Chapter VIII is an introduction to classical potential theory.

We will now try to highlight some of the novel features of the book: proofs or approaches that appear to be new or at least not well known and topics not usually included in books of comparable character.

Cramér’s theorem on large deviations, with an emphasis on inequalities (rather than just logarithmic asymptotics), is proved in Chapter I. Then it is used to provide the tail estimates needed to prove the Hartman–Wintner law of the iterated logarithm for bounded random variables. The general finite variance case is reduced to the bounded case using ideas originating in the law of the iterated logarithm in Banach spaces (see Ledoux and Talagrand [4]; for another contemporary elementary proof, see Griffin and Kuelbs [3]).

In Chapter II the Berry–Esséen theorem is proved using Bolthausen’s adaptation of Stein’s method [5], rather than the classical Fourier-analytic technique. Section 2.3 contains the first analytical excursion. The central limit theorem is used to prove a criterion for the hypercontractivity of Hermite multipliers [if μ is a probability measure and $1 \leq p \leq q$, then a linear operator of norm 1 from $L^p(\mu)$ into $L^q(\mu)$ is said to be hypercontract-