

and $\lambda_2(R) = 4\lambda_2$, the mean error is easily found. Thus the squares of the mean errors of the co-ordinates x and y are

$$\begin{aligned}\lambda_2(x) &= \{2.3490(0.4257 + 0.0228 \cos V)^2 + 1.2943(-0.1752 \cos V)^2 + 4(0.25 \cos V)^2\} \lambda_2, \\ \lambda_2(y) &= \{2.3490(-0.1444 + 0.0228 \sin V)^2 + 1.2943(0.7726 - 0.1752 \sin V)^2 + 4(0.25 \sin V)^2\} \lambda_2.\end{aligned}$$

Only the value $\lambda_2 = 0.00236$, found by the summary criticism, is here very uncertain.

XIII. SPECIAL AUXILIARY METHODS.

§ 57. We have often occasion to use the method of least squares, particularly adjustment by elements; and this sometimes requires so much work that we must try to shorten it as much as possible, even by means which are not quite lawful. Several temptations lie near enough to tempt the many who are soon tired by a somewhat lengthened computation, but not so much by looking for subtleties and short cuts. And as, moreover, the method was formerly considered the best solution — among other more or less good — not the only one that was justified under the given supposition, it is no wonder that it has come to be used in many modifications which must be regarded as unsafe or wrong. After what we have seen of the difference between free and bound functions, it will be understood that the consequences of transgressions against the method of least squares stand out much more clearly in the mean errors of the results than in their adjusted values. And as — to some extent justly — more importance is attached to getting tolerably correct values computed for the elements, than to getting a correct idea of the uncertainty, the lax morals with respect to adjustments have taken the form of an assertion to the effect that we can, within this domain, do almost as we like, without any great harm, especially if we take care that a sum of squares, either the correct one or another, becomes a minimum. This, of course, is wrong. In a text-book we should do more harm than good by stating all the artifices which even experienced computers have allowed themselves to employ, under special circumstances and in face of particularly great difficulties. Only a few auxiliary methods will be mentioned here, which are either quite correct or nearly so, when simple caution is observed.

§ 58. When methodic adjustment was first employed, large numbers of figures were used in the computations (logarithms with 7 decimal places), and people often complained of the great labour this caused; but it was regarded as an unavoidable evil, when the elements were to be determined with tolerable exactness. We can very often manage, however, to get on by means of a much simpler apparatus, if we do not seek something