shall discuss these two forms in the following chapters XI and XII, first the adjustment by correlates whose rules it is easiest to deduce. In practice we prefer adjustment by correlates when m is nearly as large as n, adjustment by elements when m is small.

XI. ADJUSTMENT BY CORRELATES.

§ 47. We suppose we have ascertained that the whole theory is expressed in the equations $[au] = A, \dots [cu] = C$, where the adjusted values u of the n observations are the only unknown quantities; we prefer in doubtful cases to have too many equations rather than too few, and occasionally a supernumerary equation to check the computation. The first thing the adjustment by correlates then requires is that the functions $[ao] \dots [co]$, corresponding to these equations, are made free of one another by the schedule in § 42.

Let $[ao], \dots [c''o]$ indicate the $n-m$ mutually free functions which we have got by this operation, and let us, beside these, imagine the system of free functions completed by m other arbitrarily selected functions, $[d'''o], \dots [g^{\nu}o]$, representatives of the empiric functions; the adjustment is then principally made by introducing the theoretical values into this system of free functions. It is finally accomplished by transforming back from the free modified functions to the adjusted observations. For this inverse transformation, according to (62), the n equations are:

$$o_i = \left\{\frac{a_i}{[aa\lambda_2]}[ao] + \dots + \frac{c''_i}{[c''c''\lambda_2]}[c''o] + \frac{d'''_i}{[d'''d'''\lambda_2]}[d'''o] + \dots + \frac{g^{\nu}_i}{[g^{\nu}g^{\nu}\lambda_2]}[g^{\nu}o]\right\}\lambda_2(o_i) \quad (73)$$

and according to (35) (compare also (63))

$$\left.\begin{aligned}\lambda_2(o_i) &= \left\{\frac{a_i^2\lambda_2^2(o_i)}{[aa\lambda_2]^2}\cdot\lambda_2[ao] + \dots\dots + \frac{g^{\nu 2}_i\lambda_2^2(o_i)}{[g^{\nu}g^{\nu}\lambda_2]^2}\lambda_2[g^{\nu}o]\right\} \\ &= \left\{\frac{a_i^2}{[aa\lambda_2]} + \dots + \frac{c''^2_i}{[c''c''\lambda_2]} + \frac{d'''^2_i}{[d'''d'''\lambda_2]} + \dots + \frac{g^{\nu 2}_i}{[g^{\nu}g^{\nu}\lambda_2]}\right\}\lambda_2^2(o_i)\end{aligned}\right\} \quad (74)$$

As the adjustment influences only the $n-m$ first terms of each of these equations, we have, because $[au] = A, \dots [c''u] = C''$, and $\lambda_2[au] = \dots = \lambda_2[c''u] = 0$,

$$u_i = \left\{\frac{a_i}{[aa\lambda_2]}A + \dots + \frac{c''_i}{[c''c''\lambda_2]}C'' + \frac{d'''_i}{[d'''d'''\lambda_2]}[d'''o] + \dots + \frac{g^{\nu}_i}{[g^{\nu}g^{\nu}\lambda_2]}[g^{\nu}o]\right\}\lambda_2(o_i) \quad (75)$$

and

$$\lambda_2(u_i) = \left\{\frac{d'''^2_i}{[d'''d'''\lambda_2]} + \dots + \frac{g^{\nu 2}_i}{[g^{\nu}g^{\nu}\lambda_2]}\right\}\lambda_2^2(o_i). \quad (76)$$