way. In what follows the empirical side will be treated exclusively, and it will be treated on a general plan, investigating not the particular way in which statistical, chemical, physical, and astronomical observations are made, but the common rules according to which they are all submitted to computation.

## II. LAWS OF ERRORS.

§ 5. Every observation is supposed to contain information, partly as to the phenomenon in which we are particularly interested, partly as to all the circumstances, connected with it, which are regarded as essential. In comparing several observations, it makes a very great difference, whether such essential circumstances have remained unchanged, or whether one or several of them have changed between one observation and another. The treatment of the former case, that of repetitions, is far simpler than that of the latter, and is therefore more particularly the subject of our investigations; nevertheless, we must try to master also the more difficult general case in its simplest forms, which force themselves upon us in most of the empirical sciences.

By repetitions then we understand those observations, in which all the essential circumstances remain unchanged, in which therefore the results or phenomena should agree, if all the operative causes had been included among our essential circumstances. Furthermore, we can without hesitation treat as repetitions those observations, in which we assume that no essential circumstance has changed, but do not know for certain that there has been no such change. Strictly speaking, this would furnish an example of observations with systematic errors; but provided there has been no change in the care with which the essential circumstances have been determined or checked, it is permissible to employ the simpler treatment applicable to the case of repetitions. This would not however be permissible, if, for instance, the observer during the repetitions has perceived any uncertainty in the records of a circumstance, and therefore paid greater attention to the following repetitions.

§ 6. The special features of the observations, and in particular their degree of accuracy, depend on causes which have been left out as unessential circumstances, or on some overlooked uncertainty in the statement of the essential circumstances. Consequently no speculation can indicate to us the accuracy and particularities of observations. These must be estimated by comparison of the observations with each other, but only in the case of repetitions can this estimate be undertaken directly and without some preliminary work. The phrase law of errors is used as a general name for any mathematical expression representing the distribution of the varying results of repetitions.