ON A PROPERTY OF THE SEMI-INVARIANTS OF THIELE

By

CECIL C. CRAIG
National Research Fellow

Given a general linear form

$$(1) \qquad a_1 x_1 + a_2 x_2 + \cdots\cdots + a_n x_n$$

of a set of statistical variables, $x_1, x_2, \cdots\cdots, x_n$,[1] it is well-known that in case the variables, $x_1, x_2, \cdots\cdots, x_n$ are independent, in the sense of the theory of probability, that the r'th semi-invariant of this form is simply

$$(2) \qquad a_1^r \lambda_r^{(1)} + a_2^r \lambda_r^{(2)} + \cdots\cdots + a_n^r \lambda_r^{(n)},$$[2]

in which $\lambda_r^{(i)}$ is the r'th semi-invariant of x_i. This is perhaps the most important and useful property of semi-invariants.

Each semi-invariant is defined as a certain isobaric function of the moments of weight equal to the order of the semi-invariant. The question to which this note is devoted is whether among such isobaric functions, the property given above belongs uniquely to the semi-invariant. This problem is equivalent to another which

[1]There is no loss in generality in supposing the origin so chosen for each x_i that the constant in the form is zero.

[2]Thiele, T. N., Theory of Observations (C. & E. Layton, London, 1903) p. 39.