A GENERALIZED ERROR FUNCTION*

By

Albert Wertheimer

I. INTRODUCTION

Given a set of observed values l_i ($i = 1, 2, 3, \ldots, n$.) obtained from n observations assumed to be made on the same quantity, l, under the same conditions. We seek to determine two functions $f(P, l_i)$ and $\phi(P, l_i)$ such that

$$f(P, l_i) = 0, \qquad (i = 1, 2, 3, \ldots, n)$$

defines p as a unique value assigned to the observed quantity; and $\phi(P, l_i)dl_i$ gives to within infinitesimals of higher order the probability that if another observation is made, the observed value will lie in the interval

$$l_i \leqq l \leqq l_i \cdot dl_i.$$

Gauss determined the ϕ function to be the so-called normal error law namely,

$$\phi(P, l_i) = c e^{-h^2(P - l_i)^2}$$

on the basis of the following assumptions.

(a) The product $\prod_i \phi(P, l_i)$ is to be a maximum with respect to p. Thus

$$\sum_i \frac{\partial}{\partial p} \log \phi(P, l_i) = 0,$$

$$\sum_i \frac{\partial^2}{\partial p^2} \log \phi(P, l_i) \neq 0.$$

*Presented to the American Mathematical Society, December 28, 1931.