

# ON CORRELATION SURFACES OF SUMS WITH A CERTAIN NUMBER OF RANDOM ELEMENTS IN COMMON\*

By

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*Introduction.* The study of correlation due to a common factor has been a more or less familiar one in the literature of mathematical statistics. Kapteyn,<sup>1</sup> in an exposition of the Pearsonian coefficient of correlation, considered the correlation between two sums of normally distributed variables, the sums having  $k$  random elements in common. In 1920, Rietz<sup>2</sup> devised urn schemata which yield sums with common items involved in such a way that the correlation and regression properties can be dealt by a priori methods. In a later paper, Rietz<sup>3</sup> considered two variables, each the sum of two random drawings of elements from a continuous rectangular distribution, with one of the elements in common. Here, the emphasis was placed principally upon the description of the correlation surface. Some other aspects and extensions of this problem were brought out by Karl Pearson<sup>4</sup> in an editorial discussion of Rietz's paper.

In the literature, the theory of correlation has been discussed principally in connection with its applications. One of the objects of some of the above-mentioned papers is the establishment of a closer connection between correlation theory and abstract probability theory. Such a connection would give a more precise

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\*Presented to the American Mathematical Society, Dec. 28, 1931.

<sup>1</sup>J. C. Kapteyn, "Definition of the Correlation-Coefficient," *Monthly Notices of the Royal Astronomical Society*, Vol. 72 (1912), pp. 518-525.

<sup>2</sup>H. L. Rietz, "Urn Schemata as a Basis for the Development of Correlation Theory," *Annals of Mathematics*, Vol. 21 (1920), pp. 306-322.

<sup>3</sup>H. L. Rietz, "A Simple Non-Normal Correlation Surface," *Biometrika*, Vol. 24 (1932), pp. 288-291.

<sup>4</sup>Karl Pearson, "Professor Rietz's Problem," (Editorial), *Biometrika*, Vol. 24 (1932), pp. 290-291.