ON MULTIPLE AND PARTIAL CORRELATION COEFFICIENTS OF A CERTAIN SEQUENCE OF SUMS

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In a recent paper* the writer considered a sequence of q variables defined as follows: The first variable, x_1, is defined as the sum of n_1 values of a variable, t, drawn at random from a population characterized by a rather arbitrary continuous probability function, $f(t)$. Each succeeding variable, x_i, $(i>1)$, is defined as the sum of $k_{i-1,i}$ values of t drawn at random from the n_{i-1} values composing x_{i-1}, plus the sum of $n_i - k_{i-1,i}$ values of t drawn at random from the parent population.

For variables thus defined, it was proved that the correlation coefficient between any two consecutive sums, x_i and x_{i+1}, is independent of the probability function, $f(t)$, and is given by

$$(1) \qquad r_{x_i x_{i+1}} = \frac{k_{i,i+1}}{(n_i n_{i+1})^{1/2}}$$

It was further shown that the correlation coefficient between two variables not consecutive in the sequence is equal to the product of the respective coefficients of correlation between all intermediate pairs of consecutive variables. Thus, the coefficient of correlation between x_j and x_p, $(j<p)$, is

$$r_{x_j x_p} = r_{x_j x_{j+1}} \cdot r_{x_{j+1}, j+2} \cdots r_{x_{p-2}, p-1} \cdot r_{x_{p-1} x_p} ;$$

or, in a simpler notation,

$$(2) \qquad r_{jp} = r_{j,j+1} \cdot r_{j+1,j+2} \cdots r_{p-2,p-1} \cdot r_{p-1,p} .$$

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* On Correlation Surfaces of Sums with a Certain Number of Random Elements in Common. Annals of Mathematical Statistics, Vol. IV, pp. 103-126. May, 1933.