

MATHEMATICAL EXPECTATION OF PRODUCT MOMENTS OF SAMPLES DRAWN FROM A SET OF INFINITE POPULATIONS

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Introduction

In the second part of his investigations, "On the Mathematical Expectation of Moments of Frequency Distributions,"² Tchouproff presented a method which may be interpreted as sampling from a set of infinite univariate populations. In the present paper this method is extended to the study of moments of product moments of samples drawn from a set of infinite bivariate populations. It is also shown how this method may be extended to populations of higher order by deriving some of the simpler formulae for populations of three and four variables.

Tchouproff's method has been criticised³ because of the complicated algebra. On close examination it is found, however, that it is not the algebra which is complicated but rather the symbolism. Tchouproff introduced a great variety of symbols both in his derivations and in his results. As a consequence his work seems very intricate. If, however, the number of symbols is reduced, and the symbols themselves are simplified, which can be easily accomplished, the underlying idea of Tchouproff's method is found to be very simple.

Quite a complete study of product moments of any bivariate population has been made by Joseph Pepper in his "Studies in the Theory of Sampling."⁴ His method is essentially an extension of Church's⁵ method, in his studies of univariate populations, to bivariate populations. He does not, however, derive any generalized formulae. In the present study generalized formulae for both the first moment and the variance of product moments of any order are obtained.

It may be noted here, that all of Pepper's formulae for any infinite population can be obtained from those of the present study as special cases, by assuming that all the populations in the set are identical.

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² *Biometrika*, Vol. XXI, Dec. 1929, pp. 231-258.

³ Church, A. E. R., "On the Means and Squared Standard Deviations of Small Samples from any Population," *Biometrika*, Vol. XVIII, Nov., 1926, pp. 321-394.

⁴ *Biometrika*, Vol. XXI, Dec. 1929, pp. 231-258.

⁵ Church, A. E. R., "On the Means and Squared Standard Deviations of Small Samples from any Population," *Biometrika*, Vol. XVIII, Nov., 1926, pp. 321-394.