SOME PRACTICAL INTERPOLATION FORMULAS

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Sometimes we wish to find by means of interpolation an approximation to a particular value of w_x in the interval between the known values, w_0 and w_1. But it also might be desirable in the interval from w_0 to w_1 to interpolate several approximations to w_x at equidistant values of x. It is very important to know that a formula which might be very satisfactory to interpolate a particular value in an interval might seriously fail to be the most satisfactory formula when it is desired to interpolate several values in the same interval. The range of this paper is so limited that we only wish to find by means of interpolation several approximations to the true value of w_x in the interval from w_0 to w_1 at equidistant values of x.

One way to perform an interpolation of this sort is to use osculatory interpolation.[1] The real function of osculatory interpolation is to secure smoothness at the known points, which are sometimes called pivotal points. By roughness is meant that one or more of the successive derivatives are discontinuous at the pivotal points. Experience proves that the osculatory formulas usually secure smoothness either at the expense of labor or by a loss of accuracies over the entire range from w_0 to w_1. Frequently the function of interpolation formulas is to save labor. In many cases it appears reasonable to save labor by a loss of both smoothness and accuracy. Formulas are herein selected, without direct regard for smoothness, so as to secure the best possible compromise between a maximum of accuracy and a minimum of labor. It appears that this results in many cases in a loss of smoothness that is no more objectionable than the loss in accuracy.

The actuarial profession, while trying to perfect their methods of constructing mortality tables, have made contributions of a high order of scholarship to the theory of osculatory interpolation. But since the statistician, the astronomer, the physicist, and other scientists also have occasions to make interpolations, it seems to be very important to discuss the problem of finding the most practical methods of interpolation, not only from the special viewpoint of the actuary, but also from the general viewpoint of mathematics.

Δw_x is called the first difference of w_x, and may be defined by $\Delta w_x = w_{x+1} - w_x$.

[1] Since this paper presupposes certain knowledge on the part of the reader, it may be worth while to indicate some sources of this knowledge. The elementary parts of this knowledge can be found in any good book on finite differences. "Population Statistics and Their Compilation" by Hugh H. Wolfenden, published by the Actuarial Society of America, contains an excellent summary of osculatory interpolation. This summary indicates some valuable sources of information.