ON THE PROBLEM OF CONFIDENCE INTERVALS

By J. Neyman

When discussing my paper read before the Royal Statistical Society on 19th June, 1934, Professor Fisher said that the extension of his work concerning the fiducial argument to the case of discontinuous distributions, as presented in my paper, has been reached at a great expense: that instead of exact probability statements we get only statements in the form of inequalities.

This remark raises the question whether the disadvantage of the solution which he mentioned (the inequalities instead of equalities) results from the unsatisfactory method of approach, or whether it is connected with the nature of the problem itself.

I think that the problem is of considerable general interest. For instance it may be asked whether the confidence intervals for the binomial distribution recently published by E. S. Pearson and C. J. Clopper,[1] which correspond to the probability statements in inequalities, could be bettered.

The purpose of the present note is to show, (1) that in some exceptional cases the exact probability solution of the problem exists and that then it may easily be found by the method described in Note I of my paper;[2] (2) that in the general case of discontinuous distribution exact probability statements in the problem of confidence intervals are impossible.

In particular it will be seen that exact probability statements are impossible in the case of the binomial distribution and so that the system of confidence intervals published by Clopper and Pearson could not be bettered.

In order to avoid any possible misunderstanding I shall start by restating the problem.

We shall consider a random discontinuous variate x, capable of having one or another of a finite, or at most denumerable set of values

$$x_1,\ x_2,\ \cdots\ x_n, \qquad \qquad (1)$$

We shall assume that the frequency function, say $p\ (x \mid \theta)$, of x depends upon one parameter θ, the value of which is unknown. The problem of confidence intervals consists in ascribing to every possible value of x e.g. to x_n, $(n = 1, 2, \cdots)$ a "confidence interval," say $\theta_1(n)$ to $\theta_2(n)$ such that the probability, P, of our being correct in stating

$$\theta_1(n) \leqq \theta \leqq \theta_2(n) \qquad \qquad (2)$$

whenever we observe $x = x_n$ $(n = 1, 2, \cdots)$, is either:

[1] E. S. Pearson and C. J. Clopper: The Use of Confidence or Fiducial Limits in the Case of the Binomial. Biometrika Vol. XXVI, pp. 404–413.

[2] J. R. S. S. Vol. 97, p. 589.