ON CERTAIN COEFFICIENTS USED IN MATHEMATICAL STATISTICS

By Everett H. Larguier, S.J.

I. Introduction

(1.1) We have studied here certain coefficients arising in interpolation, numerical differentiation and integration formulas in order to establish explicit expansions for these coefficients in the form of a finite summation. Ordinarily they are obtained by means of recursion relations, which necessarily demand the building up of a complete table in order to find the desired set of coefficients. By using the methods described in this paper, we are able to calculate any desired set independent of the ones which precede it in the table. In the literature we find two other expansions of the *difference quotients of zero*, one by Jeffery[1] and one by Boole.[2] Our expansion for the *differential quotients of zero* is the same as one obtained by Jeffery,[3] however the proof is more elementary and simple.

The Bernoulli numbers also find a wide range of application in many finite integration formulas, and hence our attention was drawn to the discussion of certain coefficients which occur in the study of these functions.[4] As in the cases mentioned above these coefficients are likewise ordinarily obtained by recursion formulas, but by our expansions they may be obtained directly.

II. Difference Quotients of Zero

(2.1) It is our purpose here to show that this difference quotient of zero, $\Delta^m 0^n$, may be expressed by the following summation:

$$\Delta^m 0^n = m! \sum \left(\frac{m}{m-1}\right)^{a_{m-1}} \left(\frac{m-1}{m-2}\right)^{a_{m-2}} \cdots \left(\frac{3}{2}\right)^{a_2} \left(\frac{2}{1}\right)^{a_1} \tag{1}$$

where $a_1, a_2, \cdots, a_{m-1} = 0, 1, 2, \cdots, n - m$ and $a_1 \geqq a_2 \geqq \cdots \geqq a_{m-1} \geqq 0$. Obviously the number of terms in the summation is the number of combinations of $n - m + 1$ things taken $m - 1$ together where repetitions are allowed.

(2.2) By means of the recursion relation[5]

$$\Delta^m 0^n = m\, \Delta^m 0^{n-1} + m\, \Delta^{m-1} 0^{n-1} \tag{2}$$

[1] Henry M. Jeffery, "On a method of expressing the combinations and homogeneous products of numbers and their powers by means of differences of nothing." *Quarterly Journal of Pure and Applied Mathematics*, vol. 4 (1861), pp. 364 ff.

[2] George Boole, *A Treatise on the Calculus of Finite Differences*, (Stechert, N. Y.), p. 20.

[3] Loc. cit.

[4] Steffensen, *Interpolation* (Williams & Wilkins, Baltimore), p. 125.

[5] L. M. Milne-Thompson, *Calculus of Finite Differences*, (Macmillan), p. 36, sec. 2.53, (2).