A NEW EXPOSITION AND CHART FOR THE PEARSON SYSTEM OF FREQUENCY CURVES

By Cecil C. Craig

In the course of some years of teaching classes in mathematical statistics, the author has expanded the treatment of the Pearson system of frequency functions begun in the Handbook of Mathematical Statistics[1] into an exposition that he believes possesses marked advantages in unity, clarity, and elegance. This is accomplished by expressing the variable in standard units throughout and by making the two parameters $\alpha_3(\alpha_3^2 = \beta_1, \alpha_4 = \beta_2$ in Pearson's notation) and

$$\delta = \frac{2\alpha_4 - 3\alpha_3^2 - 6}{\alpha_4 + 3}\ ^{2}$$

fundamental in the discussion. The various formulae that arise are obtained directly and in a uniform manner and are relatively simple in form and easy to use. The criteria for the different members of the system of functions are expressed very simply in terms of α_3 and δ and the chart corresponding to the extension of the Rhind diagram given by Pearson[3] takes on a strikingly simple form.

Following the beginning made in the Handbook, the system of Pearson frequency functions are to be found among the solutions of the differential equation

$$\frac{1}{y}\frac{dy}{dt} = \frac{a - t}{b_0 + b_1 t + b_2 t^2}. \tag{1}$$

For those solutions $y = f(t)$ for which,

$$(b_0 + b_1 t + b_2 t^2)\, t^n f(t)\Big]_{t=r}^{s} = 0,$$

[1] H. L. Rietz, Editor-in-Chief; Houghton-Mifflin Co., Boston (1924). See the chapter on Frequency Curves by H. C. Carver.

[2] The notation used is that of the Handbook, loc. cit., to which reference will be frequently made. The discussion of Robert Henderson, "Frequency Curves and Moments," Transactions of the Actuarial Society of America, Vol. VIII (1904), pp. 30–41, also proceeds along very similar lines, although Professor Carver was quite unaware of it when he wrote his chapter in the Handbook. The notation of the Handbook seems preferable however.

[3] Karl Pearson: Mathematical Contributions to the Theory of Evolution, XIX. Second Supplement to a Memoir on Skew Variation; Proc. Roy. Soc., A. Vol. 216 (1916), plate opposite p. 456.