SHEPPARD'S CORRECTIONS FOR A DISCRETE VARIABLE

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In the Annals of Mathematical Statistics,[1] J. R. Abernethy gave a derivation of the corrections to eliminate the systematic errors in the moments of a discrete variable due to grouping. It is the purpose of this note to considerably shorten and simplify the derivation of these corrections by an adoption of a device used by R. A. Fisher (not published so far as I know) in the case of the ordinary Sheppard's corrections.

Let us suppose that m consecutive values of the discrete variable in question are grouped in a frequency class of width k. The m smaller intervals of width k/m go to make up the class width k, the actual points representing the m values of the variable being plotted at the centers of the sub-intervals. Now let us suppose that each of m consecutive boundary points of the sub-intervals is as likely to be chosen as a boundary point of the larger intervals as any other. Then, if x_i is the class mark of the i-th frequency class, for any *true* value, x, of the discrete variable included in this frequency class, we have

$$x_i = x + \epsilon$$

in which x and ϵ are independent variables and ϵ takes on the m values

$$-\frac{m-1}{2}\,k/m,\ -\frac{m-3}{2}\,k/m,\ \cdots,\ \frac{m-3}{2}\,k/m,\ \frac{m-1}{2}\,k/m,$$

with the equal relative frequencies $1/m$.

The moments of x_i are those calculated from the grouped frequency distribution; the problem is to express the average values of the moments of x in terms of the calculated moments and k and m. The use of moment generating functions at once leads to the desired results. Denoting the s-th moment of x_i about any origin by ν'_s, the like moment of x by μ'_s, the respective moment generating functions of the two variables by $M_{x_i}(\vartheta)$ and $M_x(\vartheta)$ respectively, we have at once

$$(1)\qquad M_{x_i}(\vartheta) = M_x(\vartheta) \sum_{\epsilon=-\frac{m-1}{2}k/m}^{\frac{m-1}{2}k/m} \frac{e^{\epsilon\vartheta}}{m},$$

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[1] "On the Elimination of Systematic Errors Due to Grouping," vol. IV (1933), pp. 263–277.