ON A GENERAL SOLUTION FOR THE PARAMETERS OF ANY FUNCTION WITH APPLICATION TO THE THEORY OF ORGANIC GROWTH

By Harry Sylvester Will

Part I

I. **The Problem Stated.** A type of problem which continually arises in the ordinary course of statistical analysis is that of determining the numerical values of the parameters of a function used to represent a series of observational data. In mathematical terminology, the problem may be stated as follows:

Given, the observational series $Y_0, Y_1, \cdots Y_{n-1}$.

Assumed, the function $y = f(x, a, b, c, \cdots)$.

To find, the numerical values of the parameters $a, b, c, \cdots$.

If the function $f(x, a, b, c, \cdots)$ is linear in the parameters, the desired solution is easily obtained by familiar methods. In cases where the function is not linear, the standard procedure is to reduce it to the linear form by expansion into Taylor's series, thus:

$$f(x, a, b, c) = f(x, a_0b_0c_0) + f_a(x, a_0b_0c_0)\cdot\Delta a + f_b(x, a_0b_0c_0)\cdot\Delta b + f_c(x, a_0b_0c_0)\cdot\Delta c, \tag{1}$$

where $a = a_0 + \Delta a$, $b = b_0 + \Delta b$, $c = c_0 + \Delta c$.

The use of this method suffers from the excessive labor involved as the number of parameters to be determined increases. In cases where satisfactory values of the first approximations $a_0b_0c_0$ are not obtainable, the solution becomes impossible. The basic difficulty arises from the consideration that the Taylor theorem requires that the increments Δa, Δb, Δc shall be very small quantities.

A method of successive approximation which makes feasible the reduction of gross errors in the corrections will, I take it, be of considerable interest to mathematical statisticians. Let us, therefore, proceed to the development of a technique which accomplishes precisely this result.

II. **The Theta Technique.** Let us begin our development with the following restatement of the technical problem involved:

Given, the observational series $Y_0, Y_1, \cdots Y_{n-1}$.

Assumed, the function $y = f(x, (a_0 + \theta_1\Delta a), (b_0 + \theta_2\Delta b), (c_0 + \theta_3\Delta c))$.

To find, the values of $\theta_1, \theta_2, \theta_3$.

In this set of relations, a_0, b_0, c_0 and Δa, Δb, Δc are known quantities; while θ_1, θ_2 and θ_3 are each assumed not to exceed ± 1 in value. It follows, therefore,