

INTERNAL AND EXTERNAL MEANS ARISING FROM THE SCALING OF FREQUENCY FUNCTIONS

BY EDWARD L. DODD

The scaling¹ of frequency functions has been discussed from the standpoint of maximum likelihood. But the likelihood criterion to be satisfied sometimes leads to a minimum likelihood; and sometimes to neither a maximum nor a minimum. Scaling will be studied in this paper with reference to the likelihood actually secured, and also with reference to the character of means obtained, whether internal or external.

SECTION 1. INTRODUCTION

It is well known that a scale obtained in a curve-fitting process is sometimes a mean. Thus, with the normal function

$$(1) \quad \frac{1}{a\sqrt{2\pi}} e^{-(x/a)^2/2},$$

if the scale a is to be obtained from measurements, x_1, x_2, \dots, x_n , we commonly accept the value

$$(2) \quad a = \left\{ \frac{1}{n} \sum x_i^2 \right\}^{1/2};$$

that is, the root-mean square of the measurements. Here, the positive value of a is naturally taken. It is called the standard deviation, and thought of as an appropriate new unit of measure.

But even with the x 's all negative, and the a taken positive, O. Chisini² considered it proper to regard a as a mean of the x 's, albeit an *external* mean. From Chisini's viewpoint, this a whether regarded as positive or negative is primarily a solution of

$$(3) \quad x_1^2 + x_2^2 + \dots + x_n^2 = a^2 + a^2 + \dots + a^2.$$

In this sum of squares, the single number a may be *substituted* for each of the x 's. Perhaps this kind of mean should be called a *substitutive* mean to distinguish it from the means of general analysis which are always internal.

¹ Fisher, R. A., "On the mathematical foundation of theoretical statistics," *Philosophical Transactions of the Royal Society of London, Series A*, Vol. 222, 309-368, (1921). See p. 338.

² Chisini, O., "Sul concetto di media," *Periodico di matematico*, Series 4, Vol. 9, 106-116, (1929).