APPLICATIONS OF TWO OSCULATORY FORMULAS

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INTRODUCTION

The main purpose of this paper is to illustrate how Mr. Jenkins' osculatory formulas[1] (A) and (B) can be applied in a convenient manner. The first section of this paper will be little more than a summary of some of the formulas contained in the other three articles. The second section will contain the applications.

I. SOME MATHEMATICS OF THE FORMULAS

The Woolhouse notation will in this paper be used to stand for the differences of u_{x+n} which represents the given values of a function. The general formulas are

$$y_x = y_0 + x\Delta y_0 + \tfrac{1}{2}x(x-1)B + \tfrac{1}{6}x(x-1)(x-\tfrac{1}{2})C; \tag{1}$$

and

$$y_x = u_0 + xa_1 + \tfrac{1}{2}x(x-1)B + \tfrac{1}{6}x(x-1)(x-\tfrac{1}{2})C. \tag{2}$$

The special formulas belonging to (2) are

$$B = b - \tfrac{1}{6}d \text{ and } C = c_1 - \tfrac{1}{6}e_1\,, \tag{A}$$

where b and d are defined by $b = \frac{1}{2}(b_0 + b_1)$ and by $d = \frac{1}{2}(d_0 + d_1)$; and

$$B = b \text{ and } C = 0. \tag{B}$$

The special formulas belonging to (1) are

$$y_0 = u_0 + \tfrac{1}{4}b_0\,, \qquad B = b, \qquad \text{and } C = 0; \tag{C}$$

and

$$y_0 = u_0 - \tfrac{1}{36}d_0\,, \qquad B = b - \tfrac{1}{6}d, \qquad \text{and } C = c_1 - \tfrac{1}{6}e_1\,. \tag{D}$$

Formula (C) is equivalent to Mr. Jenkins' formula (A). Also (D) is equivalent to his formula (B).

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[1] This paper presupposes a knowledge of three other articles. The first one by Mr. Wilmer A. Jenkins is entitled "Graduation Based on a Modification of Osculatory Interpolation," and is printed in the October 1927 issue of the Transactions of the Actuarial Society of America. The other two papers are mine. One of them is entitled "Some Practical Interpolation Formulas," and is printed in the September 1935 issue of these Annals. The other one entitled "A Family of Osculatory Formulas" is printed in the October 1935 issue of the Transactions.