THE TYPE B GRAM-CHARLIER SERIES

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While much attention has been devoted to the Type A Gram-Charlier series for the graduation of frequency curves, the Type B series has been somewhat neglected. However the numerical examples to be presented later will show that the Type B series is very useful for the graduation of skew frequency curves. Wicksell[1] has demonstrated that the Gram-Charlier series may be developed from the same law of probability which forms the basis of the Pearson system of frequency curves. Rietz[2] following Wicksell gives a derivation of the Gram-Charlier series based on the binomial $(q + p)^n$. Jordan[3] gives a method for fitting Type B based on certain orthogonal polynomials which he calls G. He uses factorial moments because of the resulting ease in finding the values of the constants.

We shall consider the Type B series for a distribution of equally distanced ordinates at non-negative values of x. We shall find the values of the first few terms of the series and shall also show how the values of later coefficients may easily be found. We write the Type B series in the form

(1) $$F(x) = c_0 + c_1\Delta\psi(x) + c_2\Delta^2\psi(x) + c_3\Delta^3\psi(x) + c_4\Delta^4\psi(x) + c_5\Delta^5\psi(x) + c_6\Delta^6\psi(x)$$

where

(2) $$\psi(x) = \frac{e^{-m}m^x}{x!}, \qquad m = \mu_1', \text{ the mean,}$$
$$\Delta\psi(x) = \psi(x) - \psi(x-1) \quad \text{for } x = 0, 1, 2, \cdots s.$$

Let $f(x)$ give the ordinates of the observed distribution of relative frequencies, so that $\Sigma f(x) = 1$. To determine the coefficients $c_0, c_1, c_2, \cdots, c_6$, we have, using the method of moments,

(3)
$$\begin{aligned}
&\Sigma[c_0\psi(x) + c_1\Delta\psi(x) + c_2\Delta^2\psi(x) + c_3\Delta^3\psi(x) + \ldots + c_6\Delta^6\psi(x)] = \Sigma f(x) = 1.\\
&\Sigma x[c_0\psi(x) + c_1\Delta\psi(x) + \ldots + c_6\Delta^6\psi(x)] = \Sigma xf(x) = m.\\
&\Sigma x^2[c_0\psi(x) + c_1\Delta\psi(x) + \ldots + c_6\Delta^6\psi(x)] = \Sigma x^2f(x) = \mu_2'.\\
&\Sigma x^3[c_0\psi(x) + \ldots + c_6\Delta^6\psi(x)] = \Sigma x^3f(x) = \mu_3'.\\
&\Sigma x^4[c_0\psi(x) + \ldots + c_6\Delta^6\psi(x)] = \Sigma x^4f(x) = \mu_4'.\\
&\Sigma x^5[c_0\psi(x) + \ldots + c_6\Delta^6\psi(x)] = \Sigma x^5f(x) = \mu_5'.\\
&\Sigma x^6[c_0\psi(x) + \ldots + c_6\Delta^6\psi(x)] = \Sigma x^6f(x) = \mu_6'.
\end{aligned}$$