CORRELATION SURFACES OF TWO OR MORE INDICES WHEN THE COMPONENTS OF THE INDICES ARE NORMALLY DISTRIBUTED

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Indices are widely used in statistical analyses.[1] In many cases incorrect conclusions are drawn because indices are not uncorrelated or independent even though all of the component variables are independent. In a previous paper[2] the distribution of an index both of whose components follow the normal law was given exactly i.e. without approximation. The purpose of the present paper is to give the simultaneous distribution of two or more indices when each of the components follow the normal law. The case for two indices will be discussed in detail and the extension to more indices will be indicated.

Let x_1, x_2, and x_3, be correlated variables each being normally distributed about their respective means m_1, m_2, m_3, with standard deviations σ_1, σ_2, σ_3, and let the correlations between the variables in pairs be represented by r_{12}, r_{13}, r_{23}. Then the simultaneous distribution of these three variables will be

$$\begin{aligned}\frac{1}{(2\pi)^{\frac{3}{2}}R^{\frac{1}{2}}\sigma_1\sigma_2\sigma_3}\exp.\ -\frac{1}{2}\frac{1}{R}\Bigg[&\frac{R_{11}(x_1-m_1)^2}{\sigma_1^2}+\frac{R_{22}(x_2-m_2)^2}{\sigma_2^2}+\frac{R_{33}(x_3-m_3)^2}{\sigma_3^2}\\ &+2R_{12}\frac{(x_1-m_1)(x_2-m_2)}{\sigma_1\sigma_2}+2R_{13}\frac{(x_1-m_1)(x_3-m_3)}{\sigma_1\sigma_3}\\ &+2R_{23}\frac{(x_2-m_2)(x_3-m_3)}{\sigma_2\sigma_3}\Bigg]dx_1\,dx_2\,dx_3\end{aligned}\tag{1}$$

where

$$R=\begin{vmatrix}1 & r_{12} & r_{13}\\ r_{12} & 1 & r_{23}\\ r_{13} & r_{23} & 1\end{vmatrix}$$

and R_{ij} are the respective second order minors of R.

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[1] Rietz, H. L. "On the Frequency Distribution of Certain Ratios," Annals of Mathematical Statistics, Vol. VII, No. 3, Sept. 1936, pp. 145–153.

[2] Baker, G. A., "Distribution of the Means Divided by the Standard Deviations of Samples From Non-homogeneous Populations," Annals of Mathematical Statistics, Feb. 1932, pp. 3–5.