VARIANCE OF A GENERAL MATCHING PROBLEM*

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Let us match two decks of cards: (A) composed of t distinct groups of s identical symbols each, and (B) a target deck composed of i_1 symbols of the first kind, i_2 of the second, etc., such that

$$i_1 + i_2 + \cdots + i_t = st = n. \tag{1}$$

It is not necessary that all the i's be different from zero.

(a) *Forming the Relative Frequency Table.* The first part of the paper is concerned with forming a 2x2-way table showing the relative frequencies of hits and misses of all pairs of cards in the target deck. The notation $\begin{matrix} i \\ 0 \end{matrix}$ indicates a miss at the ith card of the target deck, $\begin{matrix} i \\ 1 \end{matrix}$ a hit. $\begin{matrix} i \\ 0 \end{matrix} = j$ indicates a miss at the ith card, with the matching card identical to the jth target card.

Case I. *ith and jth target cards the same symbol.*

i	j	Theoretical freq.	Weighted freq.	
If 0 then —[1] 0		$n - s - 1$	$(t - 1)(n - s - 1)$	2.1
0	1	s	$(t - 1)\, s = n - s$	2.2
1	0	$n - s$	$n - s$	2.3
1	1	$s - 1$	$s - 1$	2.4
			Total $= t(n - 1)$	

(2)

But $\begin{matrix} i \\ 0 \end{matrix}$ occurs in $(t - 1)/t$ of the events. Thus we must weight 2.1 and 2.2 with a factor $(t - 1)$, giving the last column in (2).

Case II. *ith and jth target cards different*

i	j	Theoretical freq.	Weighted freq.	
If $0 = j$ then — 0		$n - s$	$n - s$	3.1
$0 = j$	1	$s - 1$	$s - 1$	3.2
$0 \neq j$	0	$n - s - 1$	$(n - s - 1)(t - 2)$	3.3
$0 \neq j$	1	s	$s(t - 2)$	3.4
1	0	$n - s - 1$	$n - s - 1$	3.5
1	1	s	s	3.6
			Total $= t(n - 1)$	

(3)

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[1] Read, 'then out of $n - 1$ times'.