ON COMBINED EXPANSIONS OF PRODUCTS OF SYMMETRIC POWER SUMS AND OF SUMS OF SYMMETRIC POWER PRODUCTS WITH APPLICATIONS TO SAMPLING (Continued)

By Paul S. Dwyer

PART II. THE FUNDAMENTALS OF SAMPLING

Introduction

We consider a population of N variates in which every individual possesses a common attribute. Let the variate x_i be the measure of such an attribute for individual i. From the N variates it is possible to form $\binom{N}{n}$ different samples where each sample consists of n variates, $n \leq N$.

Each sample has its mean, variance, etc. so that there are $\binom{N}{n}$ means, $\binom{N}{n}$ variances, etc. The fundamental sampling problem, as interpreted here, is to find the relation between the moments of the $\binom{N}{n}$ means, and the moments of the $\binom{N}{n}$ variances in terms of the moments of the moments of the universe. Numerous attempts have been made to solve this problem, but each has been restricted in some way. It is the aim of Part II to indicate an approach which is broad enough to include many of the fundamental variations.

The first chapter is devoted to a listing of criteria which should be satisfied by a theoretical development which is to be considered sufficiently general. These criteria might be applied to other statistics but the theory developed here is limited to those statistics which are moments (or functions of moments) of moments. The first chapter continues with an account of the more significant papers which have contributed to a general solution of the problem. No attempt is made to indicate a complete history, but rather there is presented a brief summary of a number of the most significant contributions.

The second chapter is devoted to definitions and notation. An attempt has been made to use conventional notation whenever it is suitable.

The third chapter deals with some of the fundamental principles which are used in the general approach. It presents a crucial part of the argument as it shows how various types of sampling problems can be reduced to Carver functions.

The last three chapters deal with specific applications to some of the simpler problems. Chapter IV discusses the case of moments of the mean of the sample.