A NOTE ON THE DERIVATION OF FORMULAE FOR MULTIPLE AND PARTIAL CORRELATION*

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1. **Multiple Correlation.** Let the measurements of N individuals on each of the n variables $x_1, x_2, \cdots, x_k, \cdots, x_n$, be expressed as relative deviates; that is, such that

$$\Sigma x_k = 0, \qquad \Sigma x_k^2 = N, \qquad k = 1, 2, 3, \cdots, n,$$

where the summations extend over the N individuals.

If values of λ_k are determined so that

$$\Sigma(x_1 - \lambda_2 x_2 - \lambda_3 x_3 - \cdots - \lambda_n x_n)^2 \text{ is a minimum,}$$

and if we let

(1) $$X_1 = \lambda_2 x_2 + \lambda_3 x_3 + \cdots + \lambda_n x_n,$$

then the multiple correlation coefficient, obtained from the regression of x_1 on the remaining $n - 1$ variables, is defined as

$$r_{1.234\cdots n} = r_{x_1 X_1}.$$

The square of the standard error of estimate of x_1 on the remaining $n - 1$ variables is defined as

$$\sigma^2_{1.234\cdots n} = \frac{1}{N}\Sigma(x_1 - X_1)^2.$$

The minimizing values for λ_k are obtained from the normal equations

(2) $$\Sigma(x_1 - X_1)x_k = 0, \qquad k = 2, 3, \cdots, n.$$

which may be written in expanded notation as,

$$\begin{aligned}
\lambda_2 + r_{23}\lambda_3 + r_{24}\lambda_4 + \cdots + r_{2n}\lambda_n &= r_{12} \\
r_{32}\lambda_2 + \lambda_3 + r_{34}\lambda_4 + \cdots + r_{3n}\lambda_n &= r_{13} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots &\cdots\cdots \\
r_{n2}\lambda_2 + r_{n3}\lambda_3 + r_{n4}\lambda_4 + \cdots + \lambda_n &= r_{1n}
\end{aligned}$$

where $r_{jk} = \frac{1}{N}\Sigma x_j x_k = r_{kj}$, $r_{jj} = 1$.

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* The notions involved in this demonstration are certainly well-known. However, the directness and simplicity of the derivations may lend some merit to their exhibition. The writer is indebted to Professor Dunham Jackson for useful advice.