FIDUCIAL DISTRIBUTIONS IN FIDUCIAL INFERENCE*

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1. **Introduction.** The essential idea involved in the method of argument now known as fiducial argument, at least in a very special case, seems to have been introduced into statistical literature by E. B. Wilson [1] in connection with the problem of inferring, from an observed relative frequency in a large sample, the true proportion or probability p associated with a given attribute. Since 1930 the ideas and terminology surrounding the fiducial method have been developed by R. A. Fisher [2, 3], J. Neyman [4, 5] and others into a system for making inferences from a sample of observations about the values of parameters which characterize the distribution of the hypothetical population from which the sample is assumed to have been drawn. The functional form of the population distribution law is assumed to be known The parameters may be means, a difference between means, variances, ranges, regression coefficients, probabilities or any other descriptive indices or combinations of indices which may be considered important in specifying the distribution function of a population. In arguing fiducially about the value of a parameter, a procedure applicable to some of the simple cases begins by the calculation from the sample of an *estimate* of the parameter in question. The values of the estimate in repeated samples of the same size will theoretically cluster "near" the true value of the parameter according to a certain distribution law which can, in general, be deduced from the functional form of the population distribution law. If the distribution of the estimate involves only the one parameter, and if, as is frequently the case, one can find a function ψ of the estimate and the parameter which has a distribution not depending on the parameter, then one is able to set up, in a rather simple manner, *fiducial limits* or a *confidence interval* for the parameter corresponding to the observed value of the estimate. The limits will depend on the particular method of calculating the estimate, the value of the estimate in the sample, and on the degree of risk of being wrong which one is willing to take in stating that the limits will include between them the value of the parameter for the population under consideration. In general the smaller the degree of risk, the wider apart will be the limits. Thus for a given pair of limits there will be an associated degree of uncertainty that the true value of the parameter is actually included between those limits. This uncertainty can be expressed by a probability α calculated from the sampling distribution of the ψ function of the parameter and estimate. Under certain conditions, one can, by simply changing variables, obtain from the ψ

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