# ON THE POWER OF THE $L_1$ TEST FOR EQUALITY OF SEVERAL VARIANCES

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The criterion $L_1$ was obtained by Neyman and Pearson[1] for testing the statistical hypothesis $H_1$ that $k$ samples, known to be from normal universes, are actually from universes with equal variances, where the means are unspecified. The test seems to be of importance, when one considers the number of experiments which are concerned with the comparison of several types of treatments. The experimenter is in most cases interested in the respective means, and it is usually assumed, in order to test for significance of the difference between sample means, that the variances of the distributions are equal. At present, significance tests for justifying this assumption are rarely applied. Because of the unsatisfactory status of the problem of testing simultaneously for means and variances, the $L_1$ test is appropriate for justifying first the assumption of equal variances before testing for the means.

Neyman and Pearson have treated the sampling distribution of $L_1$ when $H_1$ is true, and Wilks and Thompson[2] have discussed the general distribution of the criterion when $H_1$ is not true. Here we shall show that the test is unbiassed when the number of observations is the same in each sample, and is in general unbiassed in the limit, in a certain sense. In addition, values of the power function have been computed for a few selected cases, when $k$ is 2, in order to exhibit qualitatively the sharpness of the test.

Let the $i$-th sample ($i = 1, 2, \cdots, k$) of $n_i$ individuals be denoted by $\Sigma_i$ and suppose $\Sigma_i$ has been drawn at random from a normal population with mean $m_i$ (unknown) and variance $\sigma_i^2 = \dfrac{1}{A_i}$. Denote the observations of $\Sigma_i$ by $x_{ir}$ ($r = 1, 2, \cdots, n_i$). Then the criterion $L_1$ is expressible[3] in terms of the observations as follows:

$$L_1^{\frac{1}{2}n} = \frac{n^{\frac{1}{2}n} \prod_{i=1}^{k} (c_i^2)^{\frac{1}{2}n_i}}{\prod_{i=1}^{k} n_i^{\frac{1}{2}n_i} \left[ \sum_{i=1}^{k} c_i^2 \right]^{\frac{1}{2}n}} \tag{1}$$

where $n = \Sigma n_i$ and $c_i^2 = \sum_{r=1}^{n_i} (x_{ir} - \bar{x}_i)^2$. For convenience we shall let $L_1^{\frac{1}{2}n} = \lambda$.

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[1] [1], pp. 461–464.

[2] See [4]. Nayer [3], studied the Type I approximation to the criterion $L_1$ and tabulated significance limits, etc.

[3] See [1], p. 464.